MATHEMATICS

Paper 9709/11
Pure Mathematics 1 (11)

Key messages

Previous reports have highlighted that the rubric on the front cover of the question paper states 'no marks will be given for unsupported answers from a calculator.' Although most candidates have taken this message on board, there is still a significant minority who have not. To justify solutions, clear working must always be shown.

For quadratic equations, it would be necessary to show factorisation, use of the quadratic formula or completing the square as stated in the syllabus. Using calculators to solve equations and writing down only the solution is not sufficient. It is also insufficient to quote only the formula: candidates need to show values substituted into it. Factors must also always produce the coefficients of the quadratic when expanded.

Similarly, with definite integrals, a detailed method showing limits substituted must support the final answer. Using a calculator for integration is not sufficient although it can be helpful for checking.

It is always advisable to read the question carefully to see whether later parts are linked with earlier parts. In some questions it was necessary to use values found earlier.

General comments

Some very good scripts were seen, although generally the paper was found to be quite challenging. Success in this paper requires candidates to be familiar with the mathematical methods specified in the syllabus and to understand when to apply them. While some questions involved interpretation and problem-solving, several of these questions were easily accessible to candidates who had practised these methods.

Candidates should be reminded to take care with their presentation. When candidates make several attempts at a question, they should clearly indicate which solution is to be marked by crossing out unwanted working. If there is insufficient space, they should use the extra page at the end instead of writing in small spaces within another solution. Clear working will minimise the risk of making errors through misreading their own work. Some of the answers were written in pencil which is then superimposed with ink giving a very unclear image when the script is scanned. Centres are strongly advised that candidates should be told not to do this.

Comments on specific questions

Question 1

Most candidates started by rewriting $\tan\theta$ as $\dfrac{\sin\theta}{\cos\theta}$ but many could make little or no further progress. Those who found the solution often did so without factorising so could not be awarded the second M1, only the SC mark for a correct answer. Some candidates multiplied through by $\sin\theta$ to generate $\sin^2\theta$ in order to use the relevant identity. Weaker candidates used incorrect trigonometric identities such as $\sin\theta = 1 - \cos\theta$. A small number solved $\cos\theta = \dfrac{1}{4}$, leading to 75.5, or gave their answer as 104 which was not rounded to 1 decimal place, so could not be awarded the final mark.

Question 2

Candidates who were familiar with the binomial expansion generally produced accurate answers in **(a)** and **(b)**, though sign errors were seen in the second and third terms in **(b)**. Those who attempted to multiply brackets instead were prone to error. There was some confusion over which terms were needed, with the constant term omitted in some cases. In **(c)**, some candidates were awarded both marks, having found the expansions correctly then identifying the terms that contributed to the coefficient in x^2. Those who had incorrect expansions but successfully multiplied the correct pairs of terms were awarded M1. Weaker candidates wrote the x^2 terms from both expansions and either added them or chose the bigger coefficient.

Question 3

This is one of the new topics introduced in 2019 and there is evidence that candidates are better prepared for this type of question, with some using correct terminology in a fully correct solution. A translation should be described using a vector or a clear statement that it was translated left horizontally (or in the negative x-direction) by 6. For the stretch, it was necessary to specify the direction (in this case the y-direction or vertically) and the scale factor (2). A number of candidates misinterpreted the question and provided instructions for the reverse transformation, $g(x)$ to $f(x)$. Often there was more description present than needed which led to multiple answers, some of which were contradictory.

Question 4

Most candidates found the correct value of θ in radians and the arc length. Stronger responses used correct trigonometry, splitting triangle ABC into two right-angled triangles and using $\sin\frac{1}{12}\pi$ to find half the length of BC. An alternative method was to use the cosine or sine rule to find the length of BC. Candidates should learn these rules and use them correctly in order to be awarded the method mark. Some candidates attempted to find the area of segment BCD rather than its perimeter. Others assumed that ABC was a right-angled triangle and wrote $BC = 8\tan\theta$.

Question 5

Candidates appeared to find this question challenging, with many of them unable to proceed with a method or introducing multiple algebraic and sign errors. Successful candidates generally equated y for the line and the curve to reach $2kx^2 - 2kx + 1 = 0$ or similar, then used $b^2 - 4ac = 0$ to find k. It was common to see $-\frac{1}{2x}$ becoming $-\frac{1}{2}x$ in the working, leading to incorrect equations, in many cases not quadratic equations. Despite this, a number of candidates attempted to use the discriminant. Some candidates did not reject $k = 0$. When candidates solve a quadratic to find two possible answers, they should check that both answers make sense in the context of the question.

In the alternative method, some candidates found $\frac{dy}{dx} = \frac{1}{2x^2}$ correctly and a few of them realised $k = \frac{1}{2x^2}$ and so formed the equation $\frac{1}{2x^2}x - \frac{1}{2x^2} = -\frac{1}{2x}$ which they solved to find x. Incorrect differentiation was common, e.g. $y = 2x^{-1}$ leading to $\frac{dy}{dx} = -2x^{-2}$ or $y = (2x)^{-1}$ leading to $\frac{dy}{dx} = -(2x)^{-2}$.

Question 6

(a) Candidates who used $(2p-6) - \frac{p^2}{6} = p - (2p-6)$ or $2(2p-6) = p + \frac{p^2}{6}$ were often able to arrive at $p^2 - 18p + 72 = 0$ or an equivalent correct quadratic equation. Some candidates found both solutions but did not reject $p = 6$. Poor presentation was a handicap in this question. Many candidates made several attempts, or misread their own work, and multiple errors were seen in manipulating the algebra.

(b) Successful candidates found a, the second term and r using $p = 12$ then calculated the sum to infinity from $\frac{a}{1-r}$. A follow through mark was available for candidates who used this formula with an incorrect value of p, provided it resulted in $|r| < 1$.

Question 7

(a) Those who considered the maximum and minimum values of $\sin\theta$ could deduce that the maximum and minimum values of the overall expression were 5 and –1. A large number of candidates stated 2 as the least and 2.33 as the greatest simply by substituting lower and upper values of the domain. For some, there was some confusion between range and domain. Other candidates seemed to not understand the question: their values in **(a)** were incorrect but their graph in **(b)** showed the correct maximum and minimum for which credit was given.

(b) Most candidates sketched a sinusoidal curve but the period was often incorrect. Others supplied a non-periodic graph. Sketching graphs is a necessary skill and should be practised without first drawing up a table of values. Candidates who used their incorrect maximum and minimum values from **(a)** in a periodic graph were awarded follow through marks.

(c) The majority of candidates either attempted to solve an equation or made a guess at the number of solutions. More successful candidates used their sketch to consider intersections on the graph, concluding that there would be only one solution.

Question 8

(a) Most candidates wrote $1+\frac{2a}{7-a}=\frac{5}{2}$ but this was often followed by poor algebra leading to an incorrect value of a. A common error was omitting to multiply the 1 when multiplying through by $7-a$ to clear the denominator. This meant the method mark could not be awarded. Finding b proved to be more difficult for those who seemed unsure how a composite function is formed. Those who found f(5) first were generally more successful, though a common error was to omit the '1 +' in the formula for f(x).

(b) A small number of candidates found the correct answer of $x > 1$. Many confused the range and the domain. Others gave the answer $x > 3$, the domain of f(x). A large number gave $x \neq 1$ as their answer. Poor notation was often seen.

(c) Successful responses swapped the variables and made y the subject, reaching a correct expression for $f^{-1}(x)$ in a few lines of working. Many algebraic errors were seen, and unclear working was a hindrance to some candidates.

Question 9

(a) The majority of candidates did not appreciate that this question about rates of change required differentiation so could be awarded no marks. Some did not attempt the question at all. Of those who did, a common error was to substitute $h = 10$ into the volume formula.

(b) Only the strongest candidates successfully used their $\frac{dh}{dt}$ from **(a)** to form an equation in h using the chain rule. After solving to find h, this value was substituted into the volume formula and the volume given to at least 3 significant figures. Many candidates did not answer this question, but those who did attempt it often did not realise differentiation was needed. The most common error was to substitute $h = 0.075$.

Question 10

(a) In this question, candidates needed to show all the steps in integrating y^2, including substituting the limits of $\frac{3}{2}$ and 1. Using the calculator for integration was not sufficient. Some fully correct solutions were seen, with the volume of the cylinder subtracted correctly and the final answer given in exact form with π included. Some candidates found the integral of y rather than y^2 while others did not attempt to find the volume of the cylinder. Errors were seen in integration, with the power reduced rather than increased, or the chain rule omitted. Some candidates appeared to be unsure whether to integrate or differentiate.

(b) A minority of candidates produced fully correct solutions assisted by a clear strategy, logical working and a diagram. Those who found the normal gradient often went on to find the equations of the normal and tangent then their intercepts with the x-axis. Some candidates found the lengths of the sides of the triangle unnecessarily, not realising the height of the triangle is 1 (the y-coordinate of B).

Some found the problem-solving aspects of this part more challenging, with many errors introduced, often through having an incorrect or no diagram. It is often advisable to sketch a new diagram rather than working with unclear annotations on the given graph. Many candidates omitted to use the chain rule when differentiating and, as with **(a)**, power errors were common.

Question 11

(a) Successful candidates used the given information to substitute $x = a$ into $\frac{dy}{dx}$ then set it equal to 0, leading to $6a^2 - 30a + 6a = 0$. From this they obtained $a = 4$. Some attempted to use the quadratic formula to solve a two-term quadratic. Other candidates did not read the question carefully and used $\frac{dy}{dx} = -15$.

(b) Most candidates found $\frac{d^2y}{dx^2} = 12x - 30$ correctly, so could be awarded the method mark. Some candidates had no a-value to substitute while others made arithmetical errors that led to the wrong conclusion about the nature of the stationary point.

(c) Many candidates integrated correctly to find $y = 2x^3 - 15x^2 + 6(\textit{their } a)x + c$ and proceeded to substitute $x = \textit{their } a$ and $y = -15$ for which they could be awarded B1 FT and M1. In this type of question, candidates should take care to write an equation correctly: $y = 2x^3 - 15x^2 + 24x + 1$ not $2x^3 - 15x^2 + 24x + 1$ to be awarded the final A mark.

(d) In this question, it was necessary to demonstrate a method of solving the quadratic from the given $\frac{dy}{dx} = 0$ leading to $x = 1$ and $x = 4$, then to reject $x = 4$. It was then straightforward to substitute $x = 1$ into the expression for y to obtain $(1, 12)$. A large number of arithmetical errors were seen in obtaining the coordinates, even from correct equations.

Question 12

(a) Most candidates found the correct equation of a circle but '= 10' or '= $\sqrt{10}$' were common. Other errors included omitting to square the brackets, swapping x and y, and sign errors.

(b) Some pleasing coordinate geometry work was seen. Many candidates successfully found the gradient of the radius using $\frac{y_2 - y_1}{x_2 - x_1}$ and then the correct equation of the radius. Others inverted the gradient or omitted to find the perpendicular gradient. Candidates who attempted to find the gradient of the tangent by differentiation were generally less successful.

(c) Many candidates used their answer to **(b)** to find the equation of circle Q. Others did not link the two parts of the question so were unable to form an equation. Most candidates did not attempt to verify the y-coordinate of the points of intersection of the two circles. Those who did often equated x^2 or y^2 from the two circle equations and solved to obtain $y = 11$. A quicker method was to substitute $y = 11$ into both circle equations to show that the intersections occurred at the same x (or x^2) value in both cases.

(d) Stronger candidates substituted the equation of their tangent into the equation of their circle then solved to obtain $x = \pm\sqrt{20}$. Some were able to find the correct exact values of y also. A number of candidates seemed to not understand the meaning of 'surd form' which is mentioned as required prior knowledge in the syllabus.

MATHEMATICS

Paper 9709/12
Pure Mathematics 1 (12)

Key messages

The syllabus for this paper changed in 2019 and new topics including circles and transformations were introduced. Many candidates seemed very confident with these new topics, but others may have required further practice on them.

The previous three reports have each highlighted that the question paper contains a statement in the rubric on the front cover that 'no marks will be given for unsupported answers from a calculator.' This message has been taken on board by most candidates, but there are still a significant minority for whom it has not. Clear working must always be shown to justify solutions. For quadratic equations, it would be necessary to show factorisation, use of the quadratic formula or completing the square as stated in the syllabus. Using calculators to solve equations and writing down only the solution is not sufficient. It is also insufficient to quote only the formula: candidates need to show values substituted into it. Factors must also produce the coefficients of the quadratic when expanded.

General comments

Many very good scripts were seen, although the paper was found to be challenging for candidates generally. Candidates seemed to have sufficient time to finish the paper. Presentation of work was mostly good, although some of the answers were initially written in pencil and then superimposed with ink giving a very unclear image when the script is scanned. Centres are strongly advised that candidates should be told not to do this.

Comments on specific questions

Question 1

This question was a good start to the paper for many candidates. The vast majority realised that integration was required although some weaker candidates either differentiated or used the equation of a straight line. The negative power was well dealt with by most as was the evaluation of the constant of integration. Some candidates did not cancel $\frac{4}{-2}$ to -2 or include '$y =$' and so did not score full marks.

Question 2

This question was also very well completed by most candidates and full marks was often awarded. The binomial expansion was generally very well understood although the coefficient of x^2 was sometimes given as $54a$ rather than $54a^2$. Candidates were confident forming and solving the resulting equation although the negative answer was sometimes lost.

Question 3

In **part (a)** most candidates were confident completing the square although the '4' was a challenge for some. In **part (b)** only a few saw the link with **part (a)**, and instead usually restarted using the discriminant. These attempts were often successful initially, but simplifying the resulting inequality correctly proved challenging for many.

Question 4

Most candidates recognised the question as a quadratic in x^3 and were able to solve it either directly, or by using a suitable substitution. A small number of candidates attempted to use *x* as a substitution for x^3 and this often then caused confusion. To avoid this, candidates should be encouraged to use a different letter in this type of question. Some candidates correctly found answers for x^3 but then incorrectly discarded the negative solution. Others found the cube root of $\frac{1}{8}$ to obtain $\pm\frac{1}{2}$. The most common error however occurred through using a calculator to solve the quadratic, rather than factorisation or other similar method. As stated in the 'Key messages', this should be discouraged.

Question 5

This question was the most successful one on the whole paper for candidates in general. A very large majority realised that definite integration was needed although a few applied the rules for differentiation instead. The $\frac{5}{2}$ proved a challenge for some but most were able to integrate successfully. Some candidates did not show the *x* values clearly substituted into their integrand and therefore were not awarded full marks.

Question 6

This question was the point where the paper started to be more challenging for many candidates**.** In **part (a)** many used the sine or cosine rule correctly but some were then unable to find the given expression. Angle *OPA* was sometimes stated to be θ or $2\pi - 2\theta$ rather than $\pi - 2\theta$. Those who used angle *AOP* with the cosine rule were usually more successful in finding the given answer although possibly the easiest method was to split the triangle in half and use the resulting right-angle triangle. It is very important that all necessary working is shown in the answer space, especially when the required answer is given in the question. In **part (b)** for the sector area most candidates seemed to realise a need to use the length of *OA* from the given answer in **part (a)**, although some used the arc length or their answer for *OA* instead. For the area of the triangle *OPA* those using right-angle triangles or angle *OPA* as $\pi - 2\theta$ were equally successful.

Question 7

Part (b) was generally well completed and **part (a)(i)** reasonably well, but the other two parts proved to be more challenging. In **part (a)(i)** most candidates followed the instruction to expand although the resulting $2\sin\theta\cos\theta$ term was sometimes missing. Good responses showed a simplification of the resulting equation to $2\sin\theta\cos\theta = 0$ and then stated that either $\sin\theta = 0$ or $\cos\theta = 0$. Sometimes only two answers were given rather than the three asked for in the question. A minority seemed to be expecting the question to be more complex than it actually was. Similarly in **part (ii)**, many responses seemed to over complicate the question rather than simply verifying which of the 3 solutions were valid for this changed equation; 30 per cent omitted it altogether. **Part (b)** was more familiar for candidates and many correct proofs were seen. In **part (c)** many candidates seemed unsure of what was required, and it was omitted by about 20 per cent. For those who did gain credit it was usually after cancelling $\sin\theta + \cos\theta - 1$ rather than taking this out as a common factor. This usually led only to two of the correct solutions. A few responses only considered $\sin\theta + \cos\theta - 1 = 0$ and some, when cancelled, obtained 0 rather than 1. To get full marks four correct answers were required.

Question 8

The response to this question was varied with many fully correct solutions seen but also many responses scoring few or no marks at all. In **part (a)** many candidates seemed to know that a reflection in the line $y = x$ was required but often they assumed that this line could be plotted by joining the origin to the point in the top right hand corner of the grid given. Weaker responses often involved reflecting the given graph in the horizontal line where it intercepted the *y*-axis. **Part (b)** was completed better with many candidates seemingly familiar with the required process and able to reach $\sin\frac{1}{4}x = \frac{y-3}{2}$. A significant number then tried to divide by $\sin\frac{1}{4}$ rather than taking the inverse sine. **Part (c)** proved challenging with many responses not showing a graph 'levelling out'. A significant number of responses also did not recognise the connection between their sketch and whether or not the function had an inverse. Many fully correct solutions were seen for **part (d)**, including the required order, but a significant number seemed to find the transformations topic a significant challenge.

Question 9

Part (a) of this question was very well done by most candidates, but **part (b)** was much more challenging for many and about 20 per cent omitted it entirely. In **part (a)** only the weakest responses did not show a formation of the two required equations from the given information and then a combination of them into a quadratic equation in one variable. Again, there was significant use of calculator functions to solve this equation and so full marks were often not scored. In **part (b)** some candidates seemed to not recognise that their answers from **part (a)** were meant to be used. Many who were able to form the *n*th terms were unable to go on to prove the given statement. Those who were successful often divided the *n*th terms and separated $(\frac{4}{5})^{n-1}$ successfully. Incorrect statements such as $20\times(\frac{4}{5})^{n-1}=16^{n-1}$ were common.

Question 10

Part (a) was reasonably well done with most candidates substituting for *y* from the equation of the tangent into the circle although a few did replace *a* with 4. Many were able to rearrange to obtain a quadratic and then find the discriminant. The resulting quadratic in *a* was usually solved correctly although, again, there was a significant amount of calculator usage. Some candidates who found **part (a)** challenging omitted **part (b)** entirely but many more realised that the two parts were independent and were more successful in the second part. Most realised that the gradient of the normal would be -2 and that it would pass through the centre of the circle, although a significant minority did extra work and found the point *P*. **Part (c)** proved to be much more challenging: 43 per cent of candidates omitted it completely and many others just repeated some of their work from **part (b)**. Successful responses tended to either replace *y* in the original equation of the circle with $-2x+c$ and follow a similar approach to **part (a)** or found the equation of the diameter parallel to the tangent and then the points where this intersected the circle.

Question 11

Similarly, to question 10, the first two parts were completed quite well but **part (c)** proved much more challenging. Most candidates were confident in differentiating the given function although some forgot to multiply by 4 and others multiplied the -1 by 4 as well. In **part (b)** many realised that $\frac{dy}{dx}$ needed to be equated to 0 and most of these were able to successfully make *x* the subject of the resulting equation. **Part (c)** relied upon candidates knowing that the gradient of a line is equivalent to the tangent of the angle that the line makes with the horizontal. Many seemed unaware of this and omitted this part or made an incorrect statement such as $\frac{dy}{dx}$ = 0 instead and were therefore unable to make any meaningful progress. Those who realised that $\frac{dy}{dx}=2$ were often then able to solve the resulting equation to find *x,* then *y,* and then the required equation of the normal.

MATHEMATICS

Paper 9709/13
Pure Mathematics 1 (13)

Key messages

The syllabus for this paper changed in 2019 and new topics including circles and transformations were introduced. Many candidates seemed very confident with these new topics, but others may have required further practice on them.

Previous reports have highlighted that the question paper contains a statement in the rubric on the front cover that 'no marks will be given for unsupported answers from a calculator.' Although this message has been taken on board by most candidates, there are still a significant minority for whom it has not. Clear working must always be shown to justify solutions. For quadratic equations, it would be necessary to show factorisation, use of the quadratic formula or completing the square as stated in the syllabus. Using calculators to solve equations and writing down only the solution is not sufficient. It is also insufficient to quote only the formula: candidates need to show values substituted into it. Factors must also always produce the coefficients of the quadratic when expanded.

Similarly, whist some calculators can give the answer to a definite integration, to gain credit for a definite integration the algebraic result and substitution of appropriate limits must be shown.

General comments

It is worthwhile reminding candidates that when providing answers to 3 significant figures, the working should be of at least 4 significant figure accuracy to ensure their final answers are of acceptable accuracy.

Most candidates appeared able to finish this paper and there was evidence that questions had been checked and altered where appropriate. In such cases candidates should ensure their alterations are consistent throughout their solution and any content that they don't want marking is clearly crossed out.

Comments on specific questions

Question 1

The transformation shown was the combination of a stretch parallel to the x-axis by scale factor 2 and a translation of $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$. Most responses correctly detailed the transformations but the standard of description was variable. Those who chose not to use a vector to describe the translation often struggled to describe it using acceptable language and the direction of the stretch and its magnitude were regularly given incorrectly.

Question 2

Those who expressed $f(x)$ in completed square form and appreciated that the minimum value of $(x-3)^2$ is zero produced many completely correct solutions, as did those who were able to show by calculus or the properties of quadratic functions that the minimum point on the curve $y = f(x)$ is $(3, c-9)$. Setting the discriminant of $f(x) - 2$ to equal less than zero also worked well although many responses that pursued this route involved setting the discriminant to equal more than zero and no further progress was made.

Question 3

(a) This question part was amongst the best answered on the paper. The expansion of this type of expression was well understood. Minor errors in dealing with combinations and powers were rare and nearly all answers were presented in the expected simplified form.

(b) With so many correct answers seen in **part (a)** the majority of candidates were able to calculate and find the required terms in x and $\frac{1}{x}$ then obtain the required equations in *a* and *b*. When correctly obtained these were almost always solved correctly.

Question 4

(a) Several approaches were seen, with the most popular being to express $\tan^2 x$ in terms of $\sin^2 x$ and $\cos^2 x$, multiply through by $\cos^2 x$ and apply $\sin^2 x = 1 - \cos^2 x$. There were many correct versions of the required result, although some errors in the use of brackets and powers were seen.

(b) The correct use of a quadratic equation in $\cos^2 x$ was seen in most solutions and most of these used the square roots of the only valid solution to obtain 2 angles in the required range. A significant number of answers for $\cos^2 x$ were obtained without any method being shown; as mentioned in the 'Key messages' all answers must be supported by working.

Question 5

(a) The majority of solutions successfully used substitution for *y* in the circle equation to obtain a quadratic equation in *x* whose solutions allowed *y* values to be found from the line equation. Again a significant number of answers for *x* were obtained without any method of solution shown for the quadratic equation.

(b) The use of the mid-point of *AB* as the centre of the required circle was appreciated by most candidates and, although sign errors were common, most found half the length of *AB* as the required radius. The standard equation of a circle, $(x-a)^2 + (y-b)^2 = r^2$, was nearly always used. It was expected that r^2 would not be left in surd form in the final answer.

Question 6

(a) There were many excellent solutions presented for this question, with most candidates using the correct formulae for arc length and sector area. Those who chose to use $\frac{1}{2}ab\sin C$ to find the area of triangle *OAB* or the formula for segment area generally produced more concise, accurate solutions than those who used $\frac{1}{2}bh$ for the area of the triangle. In these cases, calculations tended to lose the accuracy necessary to be able to give the final answer to 3 significant figures. Many candidates were confident in using radians and few instances were seen where angles in degree measure were used. Although many candidates used correct methods in this question, final accuracy marks were often not awarded as final answers were not written to 3 significant figures.

(b) Many correct methods were seen leading to the length *AB*. Occasionally this had been found in **part (a)** and was simply quoted here. Achieving 3 significant figure accuracy was less of a problem here and more correct answers were seen.

Question 7

(a) Given the form of the function and the domain it was surprising to see only around half of candidates finding and expressing the range correctly.

(b) The change of subject method was universally favoured by candidates and most were able to reach a correct inverse function without sign or algebraic errors. Using their range from **part (a)** correctly expressed as a domain for this part was sufficient and often seen.

(c) The formation of a composite function and the subsequent combination of algebraic fractions was nearly always presented correctly. There were very few instances of the wrong order of functions or errors in algebraic manipulation and this was the most successfully answered question part on the paper.

Question 8

(a) The relationship between the terms of a geometric progression appeared to be well understood. Calculation of the common ratio in terms of the first term and substitution into the formula for the sum to infinity was the most frequently used method with accurate algebraic manipulation often leading to a correct quadratic or cubic equation. A visible method of solution for these equations was required together with selection of the positive solution for full credit.

(b) The process of obtaining the common difference from the first two terms appeared to be well understood as did the use of the formulae for the sum to n terms of an arithmetic progression. Here it was more usual for candidates to work with an equality rather than an inequality. Whilst most solutions showed the algebra used to reach a quadratic expression very few showed how this expression was solved to obtain values of n. Those who had preserved the inequality were able to show the appropriate integer solution which was apparent from their positive solution.

Question 9

(a) The need to integrate and the process of integration and the calculation of the constant of integration were well understood and applied. Most attempts were successful with few errors reported in the manipulation of the negative power and the calculation of the constant of integration.

(b) The better answers justified why the tangent gradient at $x = 0$ was 3 and why the equation of the tangent was $y = 3x + 3$. The tangent was then, almost always, equated to the candidate's equation of the curve. Whilst the required result was then only three algebraic steps away the quickest route was often missed. Some found the value of x which satisfied their equation and showed this then satisfied the given equation.

(c) Solutions which showed a full calculation of the left-hand side of the equation when $x = \frac{3}{2}$ made more progress with this part. The value of y at $x = \frac{3}{2}$ was most easily found from the tangent and this was usually seen in the better answers.

Question 10

(a) The procedure for finding the turning points on a curve was well understood and many completely correct solutions to this part were seen. Common errors were sign errors and mistakes in squaring. The differentiation of this type of function did not seem to cause undue problems for candidates.

(b) The quickest verification came from finding the gradient of the curve at A and the gradient of AB and showing they had a product of -1. Some candidates chose to find one of the gradients, predict the other using $m_1 m_2 = -1$ and then verify it by calculation of the second gradient. When explained clearly this was given full credit.

(c) This proved to be a very accessible final question part, approached equally successfully by finding the area under the curve and subtracting the area under the line or by subtracting the area of the equivalent trapezium. Correct forms of the required integration of the curve equation were often seen and these were sometimes combined with the integration of the line equation. Only answers which showed how the limits of integration were applied gained full credit. Stating the required integration and then quoting an answer from a calculator without showing any of the integration and substitution processes gained no credit.

MATHEMATICS

Paper 9709/21
Pure Mathematics 2 (21)

Key messages

It is important that candidates check that they are giving their answers in the form specified in the question and to the correct level of accuracy if appropriate.

Each question should be read carefully to ensure that all the relevant facts have been taken into account and that the demands of the question have been met.

Candidates should also ensure that they are familiar with the rubric on the front of the examination paper.

General comments

There did not appear to be any timing issues and most candidates had more than sufficient space in which to write their answers.

It was evident that many candidates had prepared for the examination by revision and full coverage of the syllabus, although some had prepared less as evidenced by low marks obtained.

Comments on specific questions

Question 1

Most candidates attempted to use logarithms with varying levels of success. Although many responses involved taking logarithms of both sides, brackets were frequently omitted, which then lead to errors in the subsequent attempts at simplification. Errors with the application of the laws of logarithms were also common.

Question 2

It was essential that differentiation of a quotient or equivalent product was attempted to make any progress. Again, poor use of brackets led to errors in the calculation of the numerical value of the gradient. Most candidates were able to attempt the equation of the tangent using either their incorrect gradient or the correct gradient and the given point. It is important that candidates check that they are giving their answers in the required form. Too many candidates obtained a correct equation for the tangent but did not simplify to integer coefficients and subsequently the form specified in the question.

Question 3

(a) Most candidates were able to make a reasonable attempt at the integration and subsequent substitution of the limits. Some candidates did have difficulties with the correct integration of e^{2x}. Even though many responses detailed the correct equation $12 = \frac{3}{2}e^{3a} - a - \frac{3}{2}$, or equivalent, most were then unable to show a rearrangement of this equation to obtain the given result. By looking at the form of the given answer, candidates should have been able to determine that there is no exponential term and so isolate the exponential term to obtain $12 + \frac{3}{2} + a = \frac{3}{2}e^{3a}$, or equivalent, as a subsequent step in the re-arrangement process.

(b) A surprisingly large number of candidates did not attempt this question at all. For the candidates that did attempt this part, most were successful, gaining full marks. It should be noted that some candidates did not give their final answer to the required level of accuracy, but most candidates did give sufficient iterations to justify their final answer.

Question 4

(a) Incorrect answers were rare as most candidates realised that they needed to use the factor formula which provided the required answer easily. Some candidates did attempt to use algebraic long division, usually with less success.

(b) It was intended that algebraic long division be used to obtain a quadratic factor. Synthetic division was also acceptable providing sufficient detail was shown. The demand of the question was to find the quotient and factorise the polynomial completely, but a significant number gave their final answer as $(x-3)(2x^2+9x+10)$, not realising that the quotient could also be factorised.

(c) Many candidates gained full marks here, although some did seem to not understand the question demand and were awarded no marks. Problems occurred when candidates equated the modulus expression to a negative root obtained in **part (b)** and obtained a solution which was not discounted. It was essential that candidates realised that the only value that *a* could take was 3. Candidates were not penalised if they used an incorrect positive value obtained in **part (b)**.

Question 5

(a) It was essential that *y* be differentiated as a product for any progress to be made and many candidates did not recognise *y* as a product. Of those candidates that did attempt differentiation of a product, many omitted the 2 from the differentiation of the trigonometric term.

(b) Although candidates realised that their answer to **part (a)** needed to be equated to zero, most derivatives were not in the correct form needed to yield an equation in $\tan 2t$ and so there were very few correct solutions. Some candidates, having obtained an equation in $\tan 2t$ did not find a negative value for *t*.

Question 6

In questions of this type, it is better for candidates to not work backwards from the given answer as often they would obtain method marks which are usually unavailable once alterations are made to match the given answer. In this case it was necessary to use the appropriate double angle formula for $4\cos^2 2x$ to obtain an expression in terms of $\cos 4x$, thus enabling integration. Unfortunately few candidates were able to do this and often the only mark which many candidates did gain was for recognising that $\dfrac{1}{\cos^2 x} = \sec^2 x$.

Question 7

(a) Many candidates were able to obtain the value of *R* correctly and also the value of α, although some candidates found this angle in radians.

(b) Many candidates recognised that they needed to use their result from **part (a)** to solve the given equation. Of the candidates using a correct method, most were able to obtain the answer of 117.7°, but the answer of 29.8° was less common as candidates did not consider using a negative angle to obtain this result. Many candidates also obtained the answer of 389.8°, but of course this was out of range.

(c) Completely correct solutions were very rare as most candidates did not relate the question to the result obtained in **part (a)**. If the result from **part (a)** was not used to write the given expression as $\dfrac{150}{25\cos\left(\dfrac{1}{2}\beta - 73.74^\circ\right) + 50}$, then little progress could be made. The question then depended upon the candidate realising that greatest possible value of V was obtained when $25\cos\left(\dfrac{1}{2}\beta - 73.74^\circ\right) = -1$. From this, the maximum value of V and the corresponding value of β could have been obtained.

MATHEMATICS

Paper 9709/22
Pure Mathematics 2 (22)

Key messages

It is important that candidates check that they are giving their answers in the form specified in the question and to the correct level of accuracy if appropriate.

Each question should be read carefully to ensure that all the relevant facts have been taken into account and that the demands of the question have been met.

Candidates should also ensure that they are familiar with the rubric on the front of the examination paper as well as the formulae that are given in the List of Formulae and Statistical Tables MF19.

General comments

There did not appear to be any timing issues and most candidates had more than sufficient space in which to write their answers.

It was evident that many candidates had prepared for the examination by revision and full coverage of the syllabus, although some had prepared less as evidenced by low marks obtained.

Comments on specific questions

Question 1

Very few completely correct responses were seen. It was important that candidates recognised that they needed to obtain an equation in terms of $\sec\theta$ or $\cos\theta$ only. It was essential that candidates expressed $5\tan^2\theta$ as $5\left(\sec^2\theta - 1\right)$. Errors included the omission of 5 and sign errors. It was also acceptable to attempt to obtain the given equation in terms of sine and cosine initially and then attempt an equation in $\cos\theta$ only. A large number of responses did not mention either the trigonometric identities, even though the relevant identity is given in the List of Formulae MF19, or the fact that $\sec\theta = \frac{1}{\cos\theta}$.

Question 2

Again very few correct responses were seen. It was essential that the given equation $y = Ae^{(A-B)x}$ be written in the form $\ln y = \ln A + (A-B)x$. Many candidates found the gradient of the straight line but were unable to relate it correctly to $A - B$. The solution of the resulting simultaneous equations also proved problematic as the term involving $(A-B)$ needed to be eliminated so that an equation in terms of $\ln A$ remained.

Substitution of the correct coordinates into $y = Ae^{(A-B)x}$ was also acceptable, but very few candidates realised that the coordinates that needed to be used were $\left(0.4, e^{3.6}\right)$ and $\left(2.9, e^{14.1}\right)$.

Question 3

It was pleasing to see that most candidates realised that an integration of $\frac{6}{2x+3}$, with respect to *x* was needed and obtained a result of the form $k\ln(2x+3)$ before the substitution of the limits 6 and zero. The fact that the question specified what form the answer needed to be given in did help some candidates realise that logarithms were involved in the integration process. Many fully correct responses were seen.

Question 4

(a) Most candidates produced a correct sketch of $y=|5x-4|$. It was essential that some indication be made of the two points of intersection with the given curve implying that there are exactly two solutions to the equation $3-e^{-\frac{1}{2}x}=|5x-4|$. A comment with words to the effect that there were two points of intersection, so two solutions, or some indication on the graph was expected.

(b) The candidates needed to consider the left-hand side of the modulus graph which was equivalent to the equation $y=4-5x$. The best way to show by calculation that α lies between 0.36 and 0.37, is to make use of the expression $3-e^{-\frac{1}{2}x}+5x-4$ or $3+e^{-\frac{1}{2}x}-5x+4$ and substitute in the values of 0.36 and 0.37. Many candidates did just this. It is essential that the resulting decimal values are shown and not just indications of the expression having a positive or negative value. It is also expected that a comment is made about the change of sign indicating that the root lies between the two value.

(c) There were many correct solutions gaining full marks to this part, but also a reasonable number of blank answer spaces. It was expected that the first value used in the iteration process was such that $0.36 \leqslant x_1 \leqslant 0.37$.

Question 5

(a) Those candidates that realised that they needed to use differentiation of a product were able to gain some marks. It was also essential that the coordinates of the point *B* were calculated. This was done correctly by most candidates that attempted this question. It should be noted that an exact value of the gradient was required.

(b) It was expected that candidates make use of their derivative from **part (a)** and equate it to zero in order to find the exact coordinates of the point *C*. Many candidates did not factorise the resulting equation, perhaps not realising that $e^{-\frac{1}{2}x}\neq 0$ and so a quadratic equation in *x* only could be obtained. Again, it was essential that exact answers be given, with some candidates having obtained the correct $x=7$, and then giving their *y* coordinate in decimal form.

Question 6

(a) In a question of this type, it is essential that each step of working be shown clearly and in full. The compound angle formulae are shown in the List of Formulae MF19, but there were still a number of responses which did not utilise these formulae. Some sign errors were made, and some candidates multiplied each bracket by 4. Use of $\sin^2\theta+\cos^2\theta=1$ and $2\sin\theta\cos\theta=\sin 2\theta$ was expected.

(b) Many correct solutions were seen making use of the given result in **part (a)**, but candidates needed to indicate that they were making a substitution of $\frac{3\pi}{8}$ or equivalent in order to gain full marks.

(c) It was expected that candidates again make use of the result from **part (a)**. Although many attempted this, a number did not realise that an angle of 2*x* rather than θ was being used.

Question 7

(a) With a given result to show, it was particularly essential that each step of working be shown clearly and in full. An attempt at the quotient rule, or equivalent product rule, in order to find $\frac{dx}{dt}$ was expected. Most candidates made correct use of $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$, but some then equated to zero rather than 1.

(b) Many correct responses were seen, with candidates realising that a substitution of $t = -1$ and use of the factor theorem was needed in order to find the value of *a*. It was not necessary that candidates had successfully attempted **part (a)** as it was expected that the given result be used.

(c) A small number of correct responses were seen for this part. Candidates were expected to factorise their expression using their value and obtain the equation $(t+1)(2t^2 + 9t + 11) = 0$, the solution of which would give the *x* values of any stationary points. The equation $(2t^2 + 9t + 11) = 0$ has no real roots and an indication of this using the discriminant or equivalent was expected.

MATHEMATICS

Paper 9709/23
Pure Mathematics 2 (23)

Key messages

It is important that candidates check that they are giving their answers in the form specified in the question and to the correct level of accuracy if appropriate.

Each question should be read carefully to ensure that all the relevant facts have been taken into account and that the demands of the question have been met.

Candidates should also ensure that they are familiar with the rubric on the front of the examination paper as well as the formulae that are given in the List of Formulae and Statistical Tables MF19.

General comments

There did not appear to be any timing issues and most candidates had more than sufficient space in which to write their answers.

It was evident that many candidates had prepared for the examination by revision and full coverage of the syllabus, although some had prepared less as evidenced by low marks obtained.

Comments on specific questions

Question 1

Very few completely correct responses were seen. It was important that candidates recognised that they needed to obtain an equation in terms of $\sec\theta$ or $\cos\theta$ only. It was essential that candidates expressed $5\tan^2\theta$ as $5\left(\sec^2\theta - 1\right)$. Errors included the omission of 5 and sign errors. It was also acceptable to attempt to obtain the given equation in terms of sine and cosine initially and then attempt an equation in $\cos\theta$ only. A large number of responses did not mention either the trigonometric identities, even though the relevant identity is given in the List of Formulae MF19, or the fact that $\sec\theta = \dfrac{1}{\cos\theta}$.

Question 2

Again very few correct responses were seen. It was essential that the given equation $y = Ae^{(A-B)x}$ be written in the form $\ln y = \ln A + (A-B)x$. Many candidates found the gradient of the straight line but were unable to relate it correctly to $A-B$. The solution of the resulting simultaneous equations also proved problematic as the term involving $(A-B)$ needed to be eliminated so that an equation in terms of $\ln A$ remained.

Substitution of the correct coordinates into $y = Ae^{(A-B)x}$ was also acceptable, but very few candidates realised that the coordinates that needed to be used were $\left(0.4, e^{3.6}\right)$ and $\left(2.9, e^{14.1}\right)$.

Question 3

It was pleasing to see that most candidates realised that an integration of $\frac{6}{2x+3}$, with respect to x was needed and obtained a result of the form $k\ln(2x+3)$ before the substitution of the limits 6 and zero. The fact that the question specified what form the answer needed to be given in did help some candidates realise that logarithms were involved in the integration process. Many fully correct responses were seen.

Question 4

(a) Most candidates produced a correct sketch of $y=|5x-4|$. It was essential that some indication be made of the two points of intersection with the given curve implying that there are exactly two solutions to the equation $3-e^{-\frac{1}{2}x}=|5x-4|$. A comment with words to the effect that there were two points of intersection, so two solutions, or some indication on the graph was expected.

(b) The candidates needed to consider the left-hand side of the modulus graph which was equivalent to the equation $y=4-5x$. The best way to show by calculation that α lies between 0.36 and 0.37, is to make use of the expression $3-e^{-\frac{1}{2}x}+5x-4$ or $3+e^{-\frac{1}{2}x}-5x+4$ and substitute in the values of 0.36 and 0.37. Many candidates did just this. It is essential that the resulting decimal values are shown and not just indications of the expression having a positive or negative value. It is also expected that a comment is made about the change of sign indicating that the root lies between the two value.

(c) There were many correct solutions gaining full marks to this part, but also a reasonable number of blank answer spaces. It was expected that the first value used in the iteration process was such that $0.36 \leqslant x_1 \leqslant 0.37$.

Question 5

(a) Those candidates that realised that they needed to use differentiation of a product were able to gain some marks. It was also essential that the coordinates of the point B were calculated. This was done correctly by most candidates that attempted this question. It should be noted that an exact value of the gradient was required.

(b) It was expected that candidates make use of their derivative from **part (a)** and equate it to zero in order to find the exact coordinates of the point C. Many candidates did not factorise the resulting equation, perhaps not realising that $e^{-\frac{1}{2}x}\neq 0$ and so a quadratic equation in x only could be obtained. Again, it was essential that exact answers be given, with some candidates having obtained the correct $x=7$, and then giving their y coordinate in decimal form.

Question 6

(a) In a question of this type, it is essential that each step of working be shown clearly and in full. The compound angle formulae are shown in the List of Formulae MF19, but there were still a number of responses which did not utilise these formulae. Some sign errors were made, and some candidates multiplied each bracket by 4. Use of $\sin^2\theta+\cos^2\theta=1$ and $2\sin\theta\cos\theta=\sin 2\theta$ was expected.

(b) Many correct solutions were seen making use of the given result in **part (a)**, but candidates needed to indicate that they were making a substitution of $\frac{3\pi}{8}$ or equivalent in order to gain full marks.

(c) It was expected that candidates again make use of the result from **part (a)**. Although many attempted this, a number did not realise that an angle of $2x$ rather than θ was being used.

Question 7

(a) With a given result to show, it was particularly essential that each step of working be shown clearly and in full. An attempt at the quotient rule, or equivalent product rule, in order to find $\frac{dx}{dt}$ was expected. Most candidates made correct use of $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$, but some then equated to zero rather than 1.

(b) Many correct responses were seen, with candidates realising that a substitution of $t = -1$ and use of the factor theorem was needed in order to find the value of *a*. It was not necessary that candidates had successfully attempted **part (a)** as it was expected that the given result be used.

(c) A small number of correct responses were seen for this part. Candidates were expected to factorise their expression using their value and obtain the equation $(t+1)\left(2t^2 + 9t + 11\right) = 0$, the solution of which would give the *x* values of any stationary points. The equation $\left(2t^2 + 9t + 11\right) = 0$ has no real roots and an indication of this using the discriminant or equivalent was expected.

MATHEMATICS

Paper 9709/31
Pure Mathematics 3 (31)

Key messages

- Read the question carefully and make sure that the answer matches the demand.
- Take care with basic algebra and arithmetic because many marks were not awarded due to basic slips.
- If a question asks for an exact answer, such as **Questions 6(b)** and **8(b)**, then a decimal approximation is not an acceptable substitute.
- In a 'show that' question, such as **Questions 4(a), 5(a), 9(a), 10(a)** and **10(b)**, take extra care with giving a full and clear explanation.
- Do not overwrite one solution with another as this makes it difficult to read when scanned.
- Your work needs to be legible – in particular, the numerals need to be clear and not ambiguous.

General comments

Some candidates demonstrated a strong understanding of the topics examined. However, for the majority, there were many blank responses. There did not appear to be any particular pattern to this – for example, candidates started **Question 6** (vectors), but about 70 percent gave no response to **part (c)**. In **Question 9**, many candidates attempted the integration by parts (**part (a)**), but more than one third of the responses to **parts (b)** and **(c)** (numerical methods) were blank. In **Question 10**, the majority of candidates attempted **part (a)** (factor theorem), but about one third offered no response to **part (b)** (on the same topic) and more than half gave no response to **part (c)**. In general, the marks were low, with a mean score of 22.6 and a quarter of all candidates were awarded fewer than 12 marks in total.

Comments on specific questions

Where numerical and other answers are given in the comments on individual questions that follow, it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct' answer.

Question 1

For the candidates who rearranged this as a quadratic equation in e^x, this was a very straight-forward question. There were many candidates who attempted to rewrite the laws of logarithms and worked through to obtain incorrect equations such as $3\ln 2x - 4\ln(-2x) = \ln 5$. In solutions with a correct initial method, the most common error was to try to process a negative value for e^x.

Question 2

(a) The sketches drawn were of varying degrees of quality. In many cases the sketch showed a curve. In sketches composed of straight lines, it was unusual for the vertex to be at $\left(-\frac{3}{2}, 0\right)$. Many had the vertex on the y-axis.

(b) **Part (a)** was intended as a hint to candidates so that they would be aware that they were only looking for a single critical value. A minority of candidates did approach this part by considering linear inequalities, but the majority chose to square the inequality to remove the modulus sign. This usually led to an incorrect critical value at $x = -5$ which formed part of the final answer.

Question 3

There were several correct responses to this question. The majority of candidates were aware of what was required for the binomial expansion. Some expanded in powers of x, rather than $4x$, and there were several slips in the arithmetic.

Question 4

(a) Many candidates made correct use of double angle formulae. The question asked candidates to form an equation in $\sin\theta$ and $\cos\theta$, but many responses did not conclude with an equation.

(b) A few candidates recognised that the equation in **part (a)** could be factorised as $(\cos\theta - \sin\theta)(\cos\theta + 3\sin\theta) = 0.$ From this form of the equation they often obtained the correct answers. Other correct solutions involved rewriting the equation as a quadratic in $\tan\theta$, or completing the square to obtain $(\cos\theta + \sin\theta)^2 = (2\sin\theta)^2$. Several candidates obtained a solvable equation by using incorrect identities.

Question 5

(a) This was a routine task that many candidates completed correctly. Several candidates treated a as a variable, so they differentiated ay^2 as a product, and several included $12a^2$ as part of their derivative. The given answers helped some candidates to identify and correct their errors.

(b) There was some confusion about how to use the information about the tangent. Some candidates used the original equation and substituted $y = 0$. Some looked for a tangent parallel to the x-axis. Several candidates obtained the correct equation $2ay = x^2$, and a minority used this correctly to find the required co-ordinates. A few solutions included incorrect points because candidates overlooked the fact that if $2ay = x^2$, then $y = -2a$ is not possible.

Question 6

(a) There were few correct solutions to this question because many candidates incorrectly assumed that $\overrightarrow{AB} = \overrightarrow{CD}$. Many appeared to be unaware of the convention that the vertices are named in sequence. Solutions were often poorly set out, with very little explanation of what the candidates were trying to do.

(b) This part of the question did not depend on the position vector of D, so most candidates started again with the given position vectors. Many demonstrated a good understanding of how to use the scalar product to find an angle. In this case the angle did not need to be found, but the exact value for the cosine of the angle was required. In several instances the only exact answer seen was $\frac{6}{3\sqrt{32}}$, which should have been simplified.

(c) The geometry required here is relatively simple: the area of the parallelogram is twice the area of triangle ABC and that can be found easily by using the formula $\frac{1}{2}ac\sin B$. The value of $\sin B$ follows from the answer to **part (b)**. Very few candidates used this approach. There were a small number of correct solutions using the perpendicular distance of a vertex from the opposite side, but the majority of candidates offered no response.

Question 7

Many solutions were awarded the first mark for the correct separation of the variables. Some recognised that the integral in y required the double angle formula and they often obtained an integral of the correct form, albeit with some errors in the coefficients. The alternative was to use integration by parts, but very few candidates completed this process correctly. There are several methods available for the integral in x. The most straight-forward was to recognise $\int \sec 2x \tan 2x \, dx$ as $k \sec 2x$. Some candidates used $\int \frac{\sin 2x}{\cos^2 2x} dx$

and integration by parts. A minority of candidates did obtain the correct forms for both integrals, but fully correct integration was unusual.

Question 8

(a) The majority of candidates demonstrated a good understanding of partial fractions. Most opted for the decomposition into three terms, but the two-term form was also accepted. The candidates who only split the fraction into two terms were not making this part any easier, and were also leaving themselves with additional work to do in **part (b)**. For candidates starting with a correct form, the most common errors were due to slips in the arithmetic.

(b) Most candidates recognised the correct forms for the integrals of some of their terms. Candidates with only two fractions often dealt with one correctly and made no progress with the second. For the logarithms, errors seen were in the signs and the coefficients. The correct answer for $\int \frac{1}{(x+2)^2}\,dx$ was less common, with many candidates giving a third log term. Several candidates did substitute the correct limits correctly, but there were few fully correct answers.

Question 9

(a) The majority of candidates demonstrated a good understanding of the process for integration by parts, and a number were awarded the first three marks. Using the limits to obtain the given equation proved to be more challenging, with several candidates not attempting to rearrange their equation.

(b) This was a familiar task, and several candidates completed it correctly. There were many possible approaches, some used the given equation, and some went back to the definite integral. Some candidates made vague statements, not supported by numerical evidence, and a large minority offered no response at all.

(c) There were a number of fully correct responses, but here again a large minority offered no response at all. For candidates using the iterative process correctly, the most common errors were due to not working to the required accuracy, or drawing the incorrect conclusion from correct work.

Question 10

(a) The majority of candidates gave a correct solution. The simplest approach was to demonstrate that $\mathrm{p}(-3)=0$. In terms of the rest of the question, dividing through by $(x+3)$ to obtain the quadratic factor $(x^2+2x+25)$ was a very helpful start.

(b) Starting from the quadratic factor it is straightforward to demonstrate the given result. Substituting $z=-1+2\sqrt{6}\mathrm{i}$ into the cubic equation and simplifying was more complicated. Using $z=-1+2\sqrt{6}\mathrm{i}$ and its conjugate to form the quadratic factor was simpler, but few candidates tried this. Candidates who used their calculators to write down the three roots of the cubic had not demonstrated that any of the values were roots of the equation and consequently were awarded no marks. There were several blank responses.

(c) Very few candidates recognised that the earlier parts of the question had been leading them to the roots of this equation. The first mark, for stating that the square roots of –3 were solutions of the equation, should have been accessible to all. Most of the candidates who made any attempt at all started by squaring $-1+2\sqrt{6}\mathrm{i}$ and made no useful progress. The majority of candidates offered no response.

MATHEMATICS

Paper 9709/32
Pure Mathematics 3 (32)

Key messages

- Read the question carefully and make sure that your answer matches the demand.
- Take care with the basic algebra and arithmetic because many marks were not awarded due to basic slips.
- If a question asks for an exact answer, then a decimal approximation is not an acceptable substitute.
- In a 'show that' question, take extra care with giving a full and clear explanation.
- Do not overwrite one solution with another as this makes it difficult to read when scanned.
- Your work needs to be legible – in particular, the numerals need to be clear and not ambiguous.

General comments

The majority of candidates showed an understanding of all the topics examined. As usual, candidates approached algebraic tasks such as **Question 1** (the inequality) and **Question 9(a)** (partial fractions) with confidence. Other topics of strength were **Question 6(c)** (iteration) and **Question 7(a) (**implicit differentiation). The trigonometry question, **Question 4**, proved to be more challenging than usual. Candidates should be reminded that in questions with a given answer, such as **Question 9(b)**, time is better spent checking and correcting errors in working rather than trying to reverse engineer an answer.

Comments on specific questions

Where numerical and other answers are given in the comments on individual questions that follow, it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct' answer.

Question 1

Most candidates demonstrated a good understanding of how to solve the inequality. There were some errors in the arithmetic, but the majority of errors were in the form of the final answer. The two common incorrect answers were $\frac{17}{11} < x < 11$ and $\frac{17}{11} > x > 11.$ Some candidates with an otherwise correct solution stated $x < \frac{17}{11}$ and $x > 11$, which is impossible.

Question 2

Those candidates who applied the laws of logarithms correctly usually obtained $x^2 = 2$. Many did not reject the negative root as being impossible. The question asks for an exact answer, so those candidates who only stated the decimal equivalent were not awarded the final mark.

Question 3

(a) The standard of most answers was very good. The majority of diagrams had a circle of the correct size in the correct place. There were a few circles of the correct size in the wrong quadrants, and in some diagrams the negative real axis was not a tangent to the circle at $(-3, 0)$. A small number of candidates did not score the final mark because they shaded the interior of their circle.

(b) The question asks for a length, but many candidates responded with an angle. There were a minority of correct answers. Some candidates recognised that they needed to find the distance from the centre of the circle to the origin, but did not then subtract the radius.

Question 4

Many candidates understood that they needed to convert the given equation to an equation in a single cosine function. A minority of candidates obtained a correct equation. The common incorrect approaches were:

– to use an incorrect 'half angle' formula thinking that $\cos\frac{1}{2}x = \frac{1}{2}\cos^2 x - 1$.
– to use the double angle formula and covert to expressions using $\cos x$ and $\cos\frac{1}{4}x$.
– to use $u = \cos\frac{1}{2}x$ and state the incorrect quadratic $2u^2 - u - 1 = 0$.
– to use the correct substitution $\cos 2x = 2\cos^2 x - 1$ but not double the formula.

Those candidates who obtained a correct quadratic equation usually solved it correctly and obtained at least one correct value for *x*. There were quite a few responses that included at least one incorrect solution in the required interval, most commonly 2π. A few used degrees rather than radians.

Question 5

(a) The minority of candidates who factorised to obtain $f(a) = (a-2)a(a+1)$ before making the substitution had a relatively simple substitution and usually reached the correct answer. Of those candidates who started by finding the square and the cube of $2 + yi$, the majority demonstrated a correct method, but there were many slips in the arithmetic and algebra. The error $(iy)^2 = -y$ was common. A few candidates showed insufficient working to make it clear that they had expanded the cube for themselves, rather than using a calculator, and these responses were not awarded full marks.

(b) The first mark was available to any candidate whose expression for $f(a)$ had a non-zero real part. Many who had the correct term, $-5y^2$, rejected the negative root, so they made no further progress. Many of those who scored the first two marks then went on to find $\arg f(a)$ rather than $\arg a$.

Question 6

(a) Successful candidates used a variety of different calculations and comparisons. The most common approach was to calculate $f(0.5)$ and $f(1)$ for $f(x) = \cot\left(\frac{1}{2}x\right) - 3x$ and note the change of sign. In some cases, candidates did not score the A mark because they did not give a clear conclusion with either a comment or suitable inequality statement.

(b) Many candidates appeared not to understand what this question was asking them to do. Several tried to apply the iterative formula to obtain numerical values rather than using the required algebraic argument. Those starting with the iterative formula and working towards $3x = \cot\left(\frac{1}{2}x\right)$ were much more likely to be successful than those who tried to work in reverse. Several candidates appeared to be confused between $\tan^{-1}\left(\frac{1}{2}x\right)$ and $\cot\left(\frac{1}{2}x\right)$.

(c) Many candidates scored full marks for this part of the question and were clearly well practised in using their calculator to carry out an iterative process. Most gave their iterations to the required 4 decimal places and the final answer to the required 2 decimal places. A minority of candidates worked in degrees, earning no marks. A common error was to round the final answer to 0.80 rather than 0.79.

Question 7

(a) This was a straight-forward question for many candidates. Some candidates did not take sufficient care in placing the minus sign, leading to ambiguous or incorrect answers. Some candidates did not use the notation for derivatives correctly.

(b) Many candidates attempted to equate the given derivative to –2 and those who did this correctly invariably obtained the correct answers. Sign errors were a problem in this part, however. The most common error was to substitute $y = -2x$ into the original equation. The question asks for exact values, so decimal approximations were not accepted.

Question 8

(a) The majority of responses were awarded the first mark for correct separation of the variables. Common errors included writing e^{2x+1} as $e^{2x} + 1$ or obtaining $\int \frac{e^{2x+1}}{dx} = \int \frac{4+9y^2}{dy}$. There were also many errors in dealing e^{-2x-1}, which often became e^{-2x+1} before integration. The majority of candidates recognised the *y* integral as requiring an inverse tangent, but many obtained an incorrect coefficient: $\frac{1}{2}$ in place of $\frac{1}{6}$ was very common. Some candidates seemed unsure about how to integrate either side of the equation and the presence of fractions in the integrands caused some to introduce logarithms to obtain expressions such as $\ln\left(4+9y^2\right)$ and $\ln\left(e^{-2x-1}\right)$. Those candidates who integrated correctly were usually able to find the constant successfully, although care should be taken when rearranging before substituting: the error of going from $tan^{-1}\left(\frac{3y}{2}\right) = -\frac{1}{2}e^{-2x-1} + c$ to $\frac{3y}{2} = \tan\left(-\frac{1}{2}e^{-2x-1}\right) + c$ was common.

(b) Candidates are becoming more proficient at this type of question. Those with a correct or nearly correct solution in **part (a)** usually gained the mark in **part (b)**. Some guessing was in evidence, with '*y* tends to zero' or '*y* tends to infinity' being common answers, and candidates are reminded that justification for their statements is required.

Question 9

(a) This seems to be a popular topic with the candidates, and many scored full marks here. A large majority chose the form of 3 fractions and fully correct solutions were common. Those candidates who found a common denominator and then substituted $x = 2$ and $x = -\frac{1}{2}$ were usually more successful than those who chose to compare coefficients, the latter method being more prone to sign errors and arithmetic slips.

A significant minority of candidates approached the square factor as a quadratic, and attempted a decomposition with $\frac{Dx+E}{(2-x)^2}$. Usually full marks was awarded in **part (a),** but then little progress was made in **(b)** as this was not in a form that could be easily integrated.

(b) This part proved to be more of a challenge and there were several errors with signs and coefficients. Many recognised that the two fractions with linear denominators integrated to log terms, but they struggled with $\int \frac{k}{(2-x)^2}\,dx$. The majority of those who completed the integration were able to substitute the limits in the correct order and some proceeded correctly to the given answer. As is often the case when an answer is given, many seemed to attempt to reverse engineer the required form from an incorrect integral: candidates should be advised that their time is better spent reviewing their solution to find an error, rather than trying to defy the laws of algebra.

Candidates who could correctly complete the integration often did not show sufficient working: candidates should be aware that a 'show that' question requires full and thorough steps of working to secure full marks.

As mentioned in **part (b)**, candidates with a fraction of the form $\frac{Dx+E}{(2-x)^2}$ often made no progress with the integration, but a few were successful in using integration by parts.

Question 10

(a) The majority of candidates recognised the need to use the product rule. This was often carried out correctly. The most common errors occurred in using the chain rule to differentiate the square root. Candidates then attempted to set the derivative equal to zero and solve for *x*. Some struggled with the two terms involving the square root. In the resulting linear equation, the common error was to obtain $-x+5$ instead of $-x-5$, resulting from an error in removing brackets. Several candidates who obtained $x=-\frac{2}{3}$ did not go on to obtain the corresponding value of *y*, or they stated *y* as a decimal. A minority of candidates avoided differentiating the square root by squaring the whole expression before attempting to differentiate.

(b) This part provided a real challenge for candidates and there were very few fully correct solutions. Many scored the B1 for the correct values for the limits for *x*, but this was commonly followed by using the limits for *u* in the wrong order. The majority of candidates substituted for all of the parts of the integrand, including correct substitution for d*x*. Errors then occurred in the attempts to tidy up the integral, which then made the subsequent integration more complicated than it needed to have been. It proved difficult for candidates to gain any further credit, either because the integration went wrong, or the limits were used the wrong way round.

Question 11

(a) Many candidates understood what they needed to do, and several scored full marks. A common error was to find the vector $\overrightarrow{AB}$ rather than the equation of the line *AB*, which then caused an absence of simultaneous equations. There were several slips in attempting to solve the simultaneous equations. A minority of candidates did all the working correctly but did not draw a clear conclusion.

(b) There were several clear and concise solutions. Early errors from some candidates who knew what was required often led to only the M1 being scored. Many candidates did not know how to get started – they often understood that a scalar product was required, but then did not use the correct vectors. There were also a large number of blank responses.

MATHEMATICS

Paper 9709/33
Pure Mathematics 3 (33)

Key messages

Candidates need to:

(i) ensure that they are prepared when they enter the examination room, that is they have a pencil, black biro, ruler, compasses and protractor. This applied particularly to **Question 3**.

(ii) be able to include the extra detail required in a proof question, such as **Question 7a**.

(iii) ensure that they retain throughout their working the independent variable given in the question, for example using θ not x **(Question 4)**, and that their handwriting clearly distinguishes between x and y (**Question 8**). This helps avoid confusion that can lead to later errors in working.

(iv) know what is meant by arg $z = -\frac{\pi}{4}$ (**Question 11**), i.e. $y = -x$, and **not** assume values such as $x = \sqrt{2}$ and $y = -\sqrt{2}$.

General comments

Generally the standard of work seen was of high quality, with only a very small number of candidates finding the paper very challenging. In addition it was good to see candidates appeared to have read recent reports and were improving their presentation skills. The earlier short questions appeared to give most candidates confidence and the time to tackle the longer later questions.

Most candidates found **Question 1** to **Question 7**, and **Question 10a** and **Question 10b**, relatively straight forward, however **Question 8**, **Question 9a** and **Question 9b**, and **Question 11a** and **Question 11b** proved more challenging.

Comments on specific questions

Question 1

Most responses were awarded full marks. The logarithms were generally removed successfully, either by subtracting first or by using exponentials, to establish a correct equation. The main error here was not giving the final answer to 3 decimal places.

Question 2

Most candidates opted to solve via long division as opposed to finding the quotient and remainder by inspection. This was usually very well done, although a few responses had the incorrect remainder by expressing the $6x$ term, after subtraction, with the incorrect sign, giving a final remainder of $-2x - 15$.

Question 3

This question was answered well by the majority of candidates. The most common error seen was in omitting a scale from the diagram or showing the scale on only one axis. Responses needed to show evidence for the centre or radius of the circle drawn. Often the second inequality was shown as a circle instead of the straight line $y = 2$, and some replaced the line $y = 2$ with $y = 4$. A few wrongly shaded the area below $y = 2$. There were a handful of freehand sketches, which were sometimes good enough to earn marks. Those who drew their diagram accurately and with correct scales, see **Key messages**, were able to obtain full marks. Candidates must draw their Argand Diagram showing equal scales on the two axes.

Question 4

This question was answered well by most candidates. Almost all were awarded the first B1, although a few missed differentiating the θ term within y. The vast majority also achieved success using either the product or the quotient rule. Again virtually all were able to obtain $\frac{\mathrm{d}y}{\mathrm{d}x}$ from their $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ and $\frac{\mathrm{d}x}{\mathrm{d}\theta}$. There were some notation errors where t or x were used instead of θ, and some omitted the independent variable completely. Candidates should ensure that, when they are asked to obtain a given answer, they take particular care to show full working and conclude with the answer printed on the question paper.

Question 5

(a) Nearly all candidates applied the product rule correctly, although there were some errors with the differential of $\cos 3x$, for instance $-\frac{1}{3}\sin 3x$ instead of $-3\sin 3x$ or $\sin x$ instead of $\sin 3x$. However, reaching the given answer took many algebraic steps, and candidates often either omitted to show that they were using $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ at $x = a$ or suggested that $\tan^{-1} 3a$ was the reciprocal of $\tan 3a$.

(b) Most candidates produced correct answers and were awarded full marks, with a very small number using degrees instead of radians. Many candidates listed more iterations than were required for convergence to the stated degree of accuracy, and some even showed a sign change as well.

Question 6

(a) The majority of candidates reached $4\cos x + \sqrt{3}\sin x$ and then continued with a fully correct answer. The main errors were either writing $-\sqrt{3}\sin x$ instead of $\sqrt{3}\sin x$ or writing $\frac{\sqrt{3}}{2}$ instead of $\sqrt{3}$. Some candidates did not replace $\cos 60$ with 0.5.

(b) There were many correct solutions seen although the second value of θ was sometimes missing or found incorrectly, for example by using $\theta = 180 - 39.2$ or $2\theta = 360 - 2 \times 39.2$. Using θ instead of 2θ or $2(\theta - 23.41)$ instead of $2\theta - 23.41$ were common errors. Occasionally degrees from **Question 6a** were combined with radians here, or vice versa.

Question 7

(a) A small number presented the proof fully correctly, with many missing dx or du or a minus sign that disappeared without the order of the limits being changed. Candidates are reminded of the need to structure carefully their answers to such questions. For example, they should change the limits and find $\frac{\mathrm{d}u}{\mathrm{d}x}$ and hence $\frac{\mathrm{d}x}{\mathrm{d}u}$ separately from the working for their integrand. Then each piece of information should be substituted singly: the expression for dx, followed by the limits and then reverse the limits due to the presence of the minus sign from differentiating $\cos x$, taking three lines of working in all. Candidates who introduced dx and reversed limits within a single line of working did not show clearly that they were carrying out all of the operations correctly.

(b) The majority of candidates answered this question well but several removed the 2 at the start and forgot to return it later. The most common errors were integrating e^{2u} incorrectly to obtain $2e^{2u}$ or making one or more sign errors. Several candidates did not spot the linkage between the two parts of the question. This meant they were faced with a far more difficult integral, and had to attempt to integrate by parts twice, often making little progress.

Question 8

There were many complete solutions seen, however a number of candidates found this question challenging and could only separate the variables correctly and integrate $\frac{1}{x}$. Of those that did recognise that the integrand in y was best dealt with by subdividing into two fractions, the majority completed the question correctly. However, some did not recognise that the integrand $\frac{4}{y^2+4}$ led to a standard arctan result and some wrote $\arctan\left(\frac{x}{2}\right)$. See **Key messages**. Some good solutions were marred by using degrees to find the constant of integration. A small number chose to complete the y integrand by parts, but after undertaking this correctly once, for which M1 was available, little or no progress was made in the next integration step. For the final mark, candidates needed to simplify their answer; $\exp^{\ln(\ldots)}$ was not sufficiently simplified as it was necessary to remove the ln.

Question 9

(a) There were many fully correct solutions, often completed efficiently in a few lines of working. However, the common error was for candidates to assume a point of intersection and solve by equating lines. The lines do meet but this is not stated in the question and should not be assumed. Often this followed work using the given point P on the line l producing $\lambda = -2$, but the fact that the intersection approach led to $\lambda = -1$, and thus what appeared to be a contradiction should have been spotted and corrected by candidates who followed this approach. The difference in the λ values arose simply because the two approaches are actually looking at different points on the line.

(b) Candidates found this part more difficult and many did not attempt it. Equating the lines was a common approach but this was only awarded full marks if the equations were checked for consistency. The most obvious and simplest approach was to create a vector from point P to a point on the line m and then to use the fact that this vector was perpendicular to the direction vector of m, hence producing the components of vectors $\overrightarrow{OQ}$ and $\overrightarrow{PQ}$, both needed in the final part of the question. Common errors were using $\frac{2}{3}$ instead of $\frac{3}{2}$ (or $\frac{3}{2}$ instead of $\frac{5}{2}$), direction errors in the relevant vectors, not squaring $\frac{3}{2}$ when using magnitudes, or finding vector $\overrightarrow{QR}$ not $\overrightarrow{OR}$. There were a number of very good correct solutions that were produced neatly within an efficient number of lines.

Question 10

(a) This question was very well done by the majority of candidates, with most scoring full marks with the correct 3 fraction format usually preferred over the 2 fraction alternative. Incorrect formats such as $\frac{A}{1+2x}+\frac{B}{3-x}$, $\frac{A}{1+2x}+\frac{C}{(3-x)^2}$ or $\frac{A}{1+2x}+\frac{B}{3-x}+\frac{C+Dx}{(3-x)^2}$ were seen. Incorrect working seen included multiplying throughout by $(1 + 2x)(3 – x)^3$ or inverting term by term to produce $A(1 + 2x) + B(3 – x) + C(3 - x)^2$. The less common approach for finding A, B and C was expanding to form simultaneous equations then solving them. This method resulted in errors more often than the substitution method.

(b) Almost all candidates obtained the first two terms of one of the relevant expansions and hence could be awarded M1. The most common errors were extracting 3^{-1} instead of 3^{-2}, extracting 3 and 9 instead of 3^{-1} and 3^{-2} or not evaluating the squared term(s) correctly. In the latter case the term was often written correctly in the initial expansion but not squared correctly when tidying up.

Question 11

(a) Most candidates recognised the need to convert to $x + \mathrm{i}y$ form but errors were often made when doing this; $3 + a^2$, $9 + a$ and $9 – a^2$ were seen in the denominator and 15 or 2 seen instead of $15a$ or $2a$ in the numerator. The correct sign was not always used in the final part and the second values of a and z were not always rejected. Some candidates opted to convert to $x + \mathrm{i}y$ and attempted to use $\pm\tan\frac{1}{4}\pi$ with arguments of $5a - 2i$ and $3 + ai$, but made little progress. Both alternative solutions were seen very infrequently.

(b) A minority of candidates attempted this part. Some responses were partially correct but used two values of z from **Question 11a**. Many used $|z|^3$ and 3θ and so achieved correct values easily. Others evaluated $(2 - 2i)^3$, not always correctly, then found the magnitude of the result. This often led to $\arg z^3 = \frac{1}{4}\pi$ from $\tan\theta = 1$. It was necessary to consider the fact that both real and imaginary parts are negative not positive.

MATHEMATICS

Paper 9709/34
Pure Mathematics 3 (34)

Key messages

- Read the question carefully and make sure that your answer matches the demand.
- Take care with the basic algebra and arithmetic because many marks were not awarded due to basic slips.
- If a question asks for an exact answer, then a decimal approximation is not an acceptable substitute.
- In a ‘show that’ question, take extra care with giving a full and clear explanation.
- Do not overwrite one solution with another as this makes it difficult to read when scanned.
- Your work needs to be legible – in particular, the numerals need to be clear and not ambiguous.

General comments

The majority of the candidates for this paper produced responses to parts of all the questions. Candidate showed confidence with the algebraic topics, such as **Question 2** (binomial expansion), **Question 7** (roots of a polynomial) and **Question 11** (if they recognised the partial fractions). In the two questions on Complex. Numbers candidates showed good skill with the Argand diagram, but they struggled with using the numbers. A sound knowledge of the basic methods in all topics was often undermined by slips in very basic algebra and arithmetic.

Candidates need to pay attention to the presentation of their solutions. The best work is clearly set out, and carefully explained. At the other extreme, some work was barely legible, standard notation was not used correctly and it was not clear what the written equations represented.

Comments on specific questions

Where numerical and other answers are given in the comments on individual questions that follow, it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only ‘correct’ answer.

Question 1

Many candidates were successful in combining the terms and removing the logarithms. The error $\ln(2x+1)+\ln(2x-1)=\ln\left(\frac{2x+1}{2x-1}\right)$ was common. The question asks for the final answer to be given to 3 decimal places, but several candidates left their answer in exact form or to 3 significant figures. Several candidates did not reject the invalid negative solution.

Question 2

Most candidates demonstrated familiarity with the binomial expansion. Many candidates found the x^3 term in the expansion of $(1+2x)^{-2}$ but did not go on to find the term in x^4. Those who did calculate the required terms were usually successful in combining them to obtain the correct answer. A small minority of candidates attempted to use partial fractions before proceeding with the expansion; this approach involved unnecessary work and errors were common. The most common error in the binomial expansion was to work with powers of *x*, rather than 2*x*. Some candidates tried to expand $(1+3x)^1$, with not all of them obtaining $1+3x$. The question asks for the coefficient of x^4 but many candidates stated the term in x^4 or the expansion up to and including the term in x^4.

Question 3

There were many fully correct solutions for this question. Common errors included using the incorrect sign in the expansion of $\cos(x-60°)$, losing the 3 in one or both terms on the right-hand side of the equation, sign errors when rearranging the equation, and errors in processing the surds. The majority of candidates knew that the graph of $\tan x$ repeats every $180°$, but $180°-x$ and $360°-x$ were common errors. Several candidates included $16.8°$ as part of their final answer.

Question 4

This was the first question that presented a real challenge to the candidates. A significant minority did not appear to understand the notation z^*, with several simply ignoring the *. Many candidates made a correct start, either simplifying the first term, or multiplying the whole equation by i. However, some did not get as far as an expression in x and y. Many candidates with an expression in x and y did not go on to form equations for the real and imaginary parts. A common error was to equate the expression to $x+\mathrm{i}y$ rather than to zero. There were many slips in the working, so fully correct solutions were unusual.

Question 5

(a) When the candidates used rulers and compasses, they often produced good diagrams. Most understood that a circle was required with a centre that involved coordinates of ±3 and ±2. Of those who attempted this part, the majority did have the correct centre. Candidates need to take care with the scales on their axes – in many cases the radius of the circle drawn did not match the scale shown. It the scales on the two axes are different, then the 'circle' should be an ellipse. The half line for $\arg z = -\frac{\pi}{4}$ did not always match the scale on the axes and did not pass through the point on the circle representing $2-2\mathrm{i}$. Several correct diagrams had the wrong region shaded or the half line drawn as a full line.

(b) A minority of responses were fully correct for this part. Some candidates did find the size of a relevant angle, but did not appreciate that it was only part of the required angle. Some candidates found the correct size for the required angle but gave a positive final answer. A minority of candidates found a length, not an angle.

Question 6

(a) Many candidates understood how to use the chain rule to obtain $\frac{\mathrm{d}y}{\mathrm{d}x}$. The work for $\frac{\mathrm{d}y}{\mathrm{d}t}$ showed that most recognised the need for the product or quotient rule. The error $\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{e}^{-t} = -t\mathrm{e}^{-t}$ was common, along with sign errors in simplifying this derivative. The work for $\frac{\mathrm{d}x}{\mathrm{d}t}$ was often incorrect, with the derivative of t sometimes being given as 0, and errors being made with the coefficients in the fraction $\frac{2}{1+4t^2}$. A significant minority did not use the correct form for the derivative of $\tan^{-1}(2t)$, often producing answers involving $\sec(2t)$ or $\sec^{-1}(2t)$. The attempts at obtaining $\frac{\mathrm{d}t}{\mathrm{d}x}$ often contained algebraic errors – a common mistake being $\frac{\mathrm{d}t}{\mathrm{d}x} = 1+\frac{1+4t^2}{2}$, or the equivalent for the candidate's derivative. The question asked for the answer to be simplified – several candidates did not get as far as collecting like terms within a bracket, and a number did not identify 'simplified' with 'factorised', making **part (b)** more difficult.

(b) A minority of responses were awarded full marks here. Many stopped when they had given a value for t. Having started with a factorised form for the derivative, several candidates expanded the numerator before trying to solve for t, and did not then obtain the correct value. Others did not give the exact value for x. Some candidates worked in degrees and obtained the incorrect final answer of 45.5.

Question 7

(a) There were many fully correct solutions to this part of the question. Most of the mistakes were due to slips in the arithmetic, although some candidates used p(–1) = 0 instead of p(–1) = 5. A few started by equating $x+1$ to 5, so used $x = 4$ and p(4) = 0.

(b) Many candidates started by dividing their p(x) by $2x + 1$. There were several candidates who obtained the correct quadratic, but did not go on to factorise it. Candidates who used a calculator to factorise the quadratic often gave the two corresponding linear factors as $\left(x-\frac{3}{2}\right)$ and $\left(x+\frac{4}{3}\right)$, making their final answer incorrect unless they included the factor 6.

Question 8

(a) This question proved challenging for the majority of candidates. Some ignored the instruction to use integration by substitution and attempted to use integration by parts. Of those who did substitute, there were several who did not attempt to use $\frac{\mathrm{d}x}{\mathrm{d}\theta}=\frac{1}{\sqrt{3}}\cos\theta.$ There were also many errors in obtaining $\frac{\sin^2\theta}{\cos^2\theta}$. Another very common error was the incorrect sequence $\left(\sqrt{1-\sin^2\theta}\right)^3=1-\sin^3\theta=\cos^3\theta.$ The justification of the new limits was often absent or incorrectly shown with several candidates writing statements involving $\sin\frac{1}{2}$.

(b) For those candidates who used the substitution $\tan^2\theta=\sec^2\theta-1$, this was a straight-forward task, and there were several concise and correct solutions. Only a minority of candidates stated the given answer without showing the full substitution of the limits. There were a few candidates who rewrote the integrand as $\sin^2\theta\sec^2\theta$ and used integration by parts – most of these attempts did not progress beyond the first stage of the integration. The most common incorrect method was to claim that $\int\tan^2\mathrm{d}\theta=\frac{1}{3}\tan^3\theta.$

Question 9

(a) There were several blank responses to this question, and many sketches were unrecognisable. The two graphs often had no labels and the axes showed either no scale or poor scales. The sketch of $y=\ln x$ tended to be the more recognisable of the two sketches shown, but it frequently crossed the x-axis at a point other than 1 and often appeared to have a horizontal asymptote or to exist for $x < 0$. The sketch of $y=1+\cot\frac{1}{2}x$ often crossed the x-axis at a value less than π. Some attempts at $\cot\frac{1}{2}x$ looked more like $\cot x$, and for some there was a possible confusion with $\operatorname{cosec}\frac{1}{2}x$ or $\sec\frac{1}{2}x$. Where a sketch showed two intersecting curves, it was unusual for the point of intersection to be highlighted as being of any importance.

(b) The majority of candidates understood what was required for this question. There were many correct approaches. Those who rewrote the equation in the form f(x) = 0 usually produced the most complete answers.

(c) Many candidates seemed to not understand the difference between this part of the question, requiring an algebraic approach, and **part (d)** requiring a numerical approach. There was also evidence of confusion between $\tan^{-1}(x)$ and $\frac{1}{\tan x}$. There were several instances of the meaningless notation $\frac{1}{\tan}(x)$. Those candidates who worked from the iterative formula towards the equation given in **part (a)** were more successful than those who worked in the other direction.

(d) The majority of candidates were able to use their calculators to complete the iterative process, with only a small minority working in degrees rather than in radians. Most candidates worked to the required degree of accuracy. This sequence converged quite slowly, and some candidates did not give sufficient iterations to confirm the value of the root.

Question 10

(a) The majority of candidates understood the structure of the vector equation of a line, but a minority did not use the correct form **r** = … . There were several slips in the arithmetic when finding $\overrightarrow{AB}$ and several candidates used the incorrect form $\mathbf{r} = \overrightarrow{OA} + \lambda\overrightarrow{OB}$. Some candidates found the vector $\overrightarrow{AB}$ but did not go on to find the equation of the line.

(b) The majority of candidates used their line and the given line correctly. There were many fully correct solutions. Most errors were due to arithmetic slips or to miscopying from one line to the next.

(c) The most successful responses to this question were the ones where it was clearly stated what the vectors being used represented – this often led to the correct vectors being used. Many of the errors were due to sign errors and to arithmetic slips. Several candidates attempted to use the scalar product of their $\overrightarrow{AP}$ with the wrong direction vector. At the final step, some candidates found the length *OP*, rather than *AP*.

Question 11

This question should have been a good source of marks for the candidates, but many found the question very challenging. The two key steps were the correct separation of variables, and to recognise that the fraction needed to be split into partial fractions. If the 25 remained with the terms in x then the numbers were simpler. Those candidates who used a decomposition into three separate fractions were usually more successful with the integration at the next stage. Those who obtained the correct partial fractions often went on to complete the question successfully. After partial fractions, the most common errors were sign errors in the course of integration. A large number of responses were awarded just one mark for correctly separating the variables and obtaining t or $\frac{1}{25}t$, as appropriate. The most common error after separation of the variables was to claim that $\int \frac{1}{5x^2 + x^3}\,\mathrm{d}x = \int \frac{1}{5x^2} + \frac{1}{x^3}\,\mathrm{d}x$.

MATHEMATICS

Paper 9709/41
Mechanics (41)

Key messages

- When answering questions involving any system of forces, a well annotated force diagram could help candidates to include all relevant terms when forming either an equilibrium situation or a Newton's law equation. Such a diagram would have been particularly useful in **Questions 5** and **6**.
- Non-exact numerical answers are required correct to three significant figures or angles correct to one decimal place as stated on the front of the question paper. Candidates are strongly advised to carry out all working to at least four significant figures if a final answer is required to three significant figures.

General comments

The questions were well answered by many candidates, and candidates at all levels were able to show their knowledge of the subject. **Questions 2(a)** and **5(a)** were found to be the easiest questions whilst **Questions 4(c), 5(b)** and **7(c)** proved to be the most challenging.

In **Question 7(c)**, the angle was given exactly as $\sin^{-1}\left(\frac{1}{60}\right)$. There is no need to evaluate the angle in this case and problems such as this can often lead to inexact answers; any approximation of the angle can lead to a loss of accuracy.

One of the rubric points on the front cover of the question paper was to take $g = 10$ and it was noted that almost all candidates followed this instruction.

Comments on specific questions

Question 1

(a) This question was answered well by most candidates. It is necessary to use the principle of conservation of linear momentum for the collision between particle P and particle Q. Most candidates used the correct form of the equation, but several candidates incorrectly either gave an answer not in terms of m or gave a negative answer. On a few occasions, conservation of kinetic energy was used instead of conservation of momentum.

(b) Most candidates gained some credit for a second application of the principle of conservation of linear momentum between particles R and Q, although a number incorrectly included particle P too. A few candidates incorrectly implied that Q continued to move after its collision with R.

Question 2

(a) This part was answered extremely well by many candidates who correctly found the greatest height above the ground reached by P. The most common error was to use $v = 10$ and $g = 10$ in $v^2 = u^2 + 2as$, which led to the 'correct' answer but from clearly incorrect working. Although rare, some candidates used an energy approach by equating the loss of potential energy $(0.4 \times g \times h)$ to the gain in kinetic energy $\left(\frac{1}{2} \times 0.4 \times 10^2\right)$.

(b) The responses to this part were mixed. Many candidates made a correct start by working out either the kinetic energy before impact (or the equivalent loss of potential energy) and then using the given value of 7.2 to work out either the speed after impact or the maximum height achieved after the first impact. The most common error when finding the time between the first and second instants at which P hit the ground was to only calculate the time from the first impact to the maximum height and so implying a time of 0.8 rather than the correct 1.6 seconds.

Question 3

This was a relatively straightforward variable acceleration question. A few candidates incorrectly integrated the given expression or attempted to use constant acceleration formulae, but most differentiated correctly and set their differentiated expression equal to zero. Many candidates struggled with solving the equation $\frac{5}{2}t^{\frac{3}{2}} - \frac{45}{8}t^{\frac{1}{2}} = 0$, with the most common errors occurring in those that attempted to square this equation; those that factorised this equation were far more successful. Some candidates, after correctly finding the time when the particle was next at rest, failed to complete the question and calculate the corresponding displacement.

Question 4

(a) This first part of the question was answered extremely well. Most candidates worked out the distance travelled by the particle in the first 10 seconds by considering the area below the line segments as two triangles and a rectangle, rather than considering the more obvious trapezium. Some candidates attempted to use the equations for constant acceleration, but these attempts were rarely successful. Where errors occurred in calculating the different areas below the line segments it was usually a slip in one (or more) value(s) or forgetting the half in the formula for the area of a triangle.

(b) It was clear in this part that many candidates failed to interact with the velocity-time graph and so were unsure how to find the minimum velocity of the particle. Even of those that did, many gave the incorrect answer of 7.2 (its speed) rather than the correct –7.2. It should also be noted that in this, as well as in **part (c)**, many candidates assumed that the two line segments between $t = 10$ and $t = T$ formed an isosceles triangle, which was not necessarily correct (but did lead to the correct answers in both these parts).

(c) Very few candidates scored full marks here and many left this part blank. The most common error, which appeared more often than the correct method, was to assume that the information given about the greatest speed of the particle was referring to the speed before time $t = 10$ and not after. Of those that did realise that the greatest speed was referring to the interval between $t = 10$ and $t = T$, many did not account for the time from 10 and implied instead that T satisfied the equation $\frac{1}{2} \times T \times 3 = 7.2$. Of those that did correctly work out that $T = 14.8$, most went on to correctly work out the average speed of the particle for the whole of its motion.

Question 5

(a) Many candidates gained most of the marks in this first part, with most not scoring full marks as they failed to correctly state the values of F and θ to at least three significant figures. Most achieved the first three marks for obtaining correct equations by resolving vertically and horizontally, although the layout was often poor.

(b) Most candidates scored the first two marks in this part for resolving vertically and horizontally and most gave the correct direction of the resultant force. However, a large number of candidates did not give the exact magnitude of the resultant force. Many gave an answer of 5.358… or left their answer as $20\sqrt{3} - 40$ without realising that this was clearly negative.

Question 6

(a) This was another question in which many candidates struggled, and it was clear that many were unsure where to begin with such a non-standard question. Many candidates did score the first two marks for correctly working out the maximum possible magnitude of the friction force at Q. Most then tried to apply either Newton's second law for the two particles either separately or for the entire system, but it was unclear at times if they knew what they were trying to achieve; the setting out of their work was often challenging to follow. The two correct approaches seen were to either consider the net force in the direction BA, which was given by the expression $0.2g\sin 60 - 0.1g\sin 60 - F_{max}$, and to indicate that this value was positive (and hence the particles were moving), or to assume that the system was in motion and show that the acceleration of the system in the direction BA would be 1.72… which again is consistent with the idea that the particles are in fact moving.

(b) Responses here were considerably better than in **part (a)**. Many applied Newton's second law correctly for P to work out the magnitude of the acceleration, before again applying Newton's second law for either the entire system or for Q only to work out the corresponding value of θ. When errors occurred, they were usually the standard sign errors or using the incorrect trigonometric ratio when resolving the weight components parallel to the two planes.

Question 7

(a) This part was answered extremely well with most candidates applying $P = F \times v$ correctly to find the driving force of the car and then using Newton's second law to find the required acceleration of the car. When errors occurred, they were usually in not using a value of 16 000 for the power, or sign errors when applying Newton's second law.

(b) Similarly to **part (a)**, this part was relatively straight-forward and therefore a well-prepared candidate did not hesitate in calculating the correct steady speed of the car.

(c) Many candidates struggled with this part. A significant number of candidates incorrectly thought that the equations for constant acceleration could be used in this problem. This is not the case as mechanically with a constant power the speed of the car is changing, hence the driving force produced by the engine of the car is variable and hence so is the car's acceleration. Furthermore, the question specifically stated that the resistance force was no longer constant. The work-energy principle had to be used here. It is necessary to find the work done by the engine, an expression for the increase in kinetic energy and the increase in potential energy. Combining these correctly with the work done by the resistive forces in the work energy equation will give an equation which can be solved to find the speed of the car at the top of the hill. Some errors were seen in trying to find a driving force from the given power and to use the given speed to find the work done by the engine. Some failed to find the correct change in height used in the potential energy calculation, and several candidates incorrectly used the change in kinetic energy as $\frac{1}{2}\times 1200\times(v-20)^2$ instead of the correct $\frac{1}{2}\times 1200\times\left(v^2-20^2\right)$.

MATHEMATICS

Paper 9709/42
Mechanics (42)

Key messages

- When answering questions involving any system of forces, a well annotated force diagram could help candidates to make sure that they include all relevant terms when forming either an equilibrium situation or a Newton's Law equation. Such a diagram would have been particularly useful here in **Questions 3**, **5, 7(a)(i)** and **7(a)(ii)**.
- Non-exact numerical answers are required correct to three significant figures as stated on the question paper. Candidates would be advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.

General comments

The requests were well answered by many candidates. Candidates at all levels were able to show their knowledge of the subject. **Questions 1, 2** and **4(a)** were found to be the most accessible questions whilst **Questions 3(a), 4(b)** and **6(d)** proved to be the most challenging.

In **Question 3(a)**, the angle α was given exactly as $\tan\alpha = \frac{4}{3}$. There is no need to evaluate the angle in situations such as this as it would not lead to the given exact answer being obtained.

One of the rubric points on the front cover of the question paper was to take $g = 10$ and it was noted that almost all candidates followed this instruction.

Comments on specific questions

Question 1

The majority of candidates earned some credit for considering either potential energy or kinetic energy or both. Those who considered both energy terms usually went on to achieve a full correct answer, with only a very small minority making sign errors. An alternative, but less successful, approach seen was to find the acceleration of the particle and then use Newton's second law to find the air resistance to the motion of the particle, subsequently multiplying this by 9 m to find the work done against air resistance.

Question 2

(a) Candidates have become increasingly confident in applying conservation of momentum to a given situation. However, a significant number did not consider that the particles were initially moving towards each other by including a negative sign with one of the initial velocities. This often resulted in a value of v as –4.5. Some subsequently made this a positive value without justification.

(b) Only a few candidates did not know or correctly apply the formula for kinetic energy here. An incorrect answer of 10.8 J was seen almost as often as the correct answer of 75.6 J. The reasons for this were twofold. Some thought that, as the request was for the loss of kinetic energy, they had to find a difference between the initial kinetic energy of each particle, rather than the difference between the total initial kinetic energy and the final kinetic energy. The second reason for this incorrect answer was for instances of the square of –6 being evaluated as –36.

Question 3

(a) This question proved a challenge for many candidates. Resolving in two directions was performed well by the majority, obtaining two correct equations in terms of P, θ and α. Only a minority of candidates did not substitute a value for α. Many candidates then used an approximate value for α such as $\alpha = 53.1°$. This created an issue of using an approximate value to obtain an exact value, which is not mathematically robust. The given $\tan\alpha = \frac{4}{3}$ should have been used so that $\sin\alpha = \frac{4}{5}$ and $\cos\alpha = \frac{3}{5}$ are substituted and then exact values are used. The other problem encountered was, as this is a 'show that' question, candidates had to show sufficient detail to convince Examiners that their method is correct. Hence those who had $P\cos\theta + 30\cos\theta = 48\cos\alpha$ immediately followed by $P\cos\theta + 30\cos\theta = 28.8$, have not shown Examiners where the 28.8 has come from, which may have been obtained from the given answer. An intermediate step of $P\cos\theta + 30\cos\theta = 48\times\frac{3}{5}$ would have been required. This is also the case for the other equation.

(b) The first request was to verify that $P = 6$. Only writing $\left(\frac{14.4}{30-P}\right)^2 + \left(\frac{28.8}{P+30}\right)^2 = 1$ is insufficient. Examiners needed to see at least one intermediate step to show verification rather than just inputting $\left(\frac{14.4}{30-P}\right)^2 + \left(\frac{28.8}{P+30}\right)^2$ into a calculator. Most candidates then went on to find θ correctly. The main error seen here was to truncate the value to 36.8, without a more accurate version being seen.

Question 4

(a) This question was answered well by most candidates, with only a very small minority giving the resistive force as $\frac{60}{3}$ and not evaluating it as 20.

(b) Almost all candidates attempted this question by using a work-energy method as was stated in the question. Although a good number of perfect solutions were seen, many solutions did not gain full credit due to prematurely approximating, particularly for the potential energy term. Others failed to gain marks by using potential energy and kinetic energy correctly, but then including the force terms 13 and 24 in their energy equation rather than the work done by these forces. There were a few candidates who used constant acceleration methods but they were rarely worthy of credit, usually due to not including the weight component in their Newton's second law equation.

Question 5

This question was well completed by many candidates. Almost all candidates resolved forces parallel and perpendicular to the inclined plane. This gave two-term expressions for both the friction, F, and the normal reaction, R. Some errors with signs were seen, as well as mixing the sine and cosine components. Some wrongly thought that the normal reaction was either $R = 0.6g\cos 35$ or $R = 0.6g$. However, candidates scored well overall on this question.

Question 6

(a) This question was answered well by many candidates.

(b) This was well attempted by most candidates who successfully differentiated the given expression for velocity and used $t = 1$ to find the required acceleration.

(c) Most knew that instantaneous rest occurs when $v = 0$ but many struggled to solve the equation which involved fractional powers. A number of candidates incorrectly believed that instantaneous rest meant solving $a = 0$ instead. Integration of the velocity to find the displacement was very well attempted by the majority of candidates.

(d) Although some very good solutions to this problem were seen by Examiners, candidates once again struggled to solve an equation with fractional powers. A surprisingly large number of candidates did not use the context of the question and incorrectly resorted to use of constant acceleration equations for this question. Many who had solved correctly went on to give the speed as a negative value.

Question 7

(a) (i) Those candidates who wrote down the two equations for the motion of P and Q separately seemed to enjoy more success than those who chose to consider the system equation, since many who attempted this method used an incorrect mass for the system. Some good answers were seen, however many candidates made errors such as omitting either the friction term or the tension in the string.

(ii) Most candidates used constant acceleration methods for this part and even those who had found an incorrect acceleration in **part (a)(i)** were still able to score the majority of the marks available. When calculating the acceleration in the section BC, many again used a system equation with the incorrect mass. A significant number of candidates did not appreciate that section BC being smooth meant that the situation is different for the rough section AB. These candidates proceeded to use a single particle equation, with the tension in the string while moving in the section AB being used for the motion in the section BC. A minority wrongly believed that the acceleration was the same for both the rough and smooth section.

(b) Some excellent answers were seen here. Even those who did not obtain full marks in the earlier parts usually used correct methods to find two times with their values.

MATHEMATICS

Paper 9709/43
Mechanics (43)

Key messages

- Non-exact numerical answers are required correct to three significant figures as stated on the question paper. Candidates would be advised to carry out all working to at least four significant figures if a final answer is required to three significant figures.
- When answering questions involving forces in equilibrium, or Newton's Second Law or an energy approach, a complete force diagram can be helpful to ensure that all relevant terms are included in the equations formed. E.g., **Question 2**, **Question 4(a),** and **Question 6(c), (d).**
- In questions with a given answer, where equations must be solved in order to find that answer, candidates are advised not to use an equation solver on their calculator since this does not explicitly 'show' the given answer. E.g., **Question 4(a)** and **Question 5(a), 5(b)**.

General comments

This paper provided the opportunity for candidates at all levels to show their knowledge of the subject, whilst providing challenge for the stronger candidates. Much work of a very high standard was seen. **Question 1**, **Question 2, Question 5(a), 5(b)** and **Question 6(a), 6(b)** were found to be the most accessible questions, whilst **Question 6(d)** and **Question 7(b)** were found to be the most challenging.

In **Question 7**, the angle was given exactly as $\sin\alpha = 0.28$. There is no need to evaluate the angle in problems such as this as any approximation of the angle can lead to a loss of accuracy in the answer.

Comments on specific questions

Question 1

This first question was a straightforward conservation of momentum problem and was well attempted by most candidates, almost all of whom found a correct equation leading to the value of $0.8\,\text{m s}^{-1}$. A few candidates made an error in solving, usually in the form of dividing the wrong way around and ending up with an answer of $1.25\,\text{m s}^{-1}$. Many candidates also found the speed when P rebounds, but some of these gave an answer of the velocity $-1.33\,\text{m s}^{-1}$ rather than the speed $1.33\,\text{m s}^{-1}$.

Question 2

This question on connected systems was again straightforward, with the acceleration, driving force, tension in the tow-bar and resistance force on the trailer given. Candidates could therefore simply apply Newton's second law to the car and then to the trailer to find the resistance force on the car F and the mass of the trailer m, with no need to use simultaneous equations. Those who took this approach almost always found the correct answers. Many candidates did not realise this and applied Newton's second law to the system, together with either the car or the trailer. Although many of these later came to the correct solutions, this approach was less successful as it was slightly more complicated, and a variety of errors were seen with this method. Some candidates had an extra term in at least one of their equations, usually including both the tension in the tow-bar and the resistance force on the trailer in the equation for the whole system.

Question 3

Many candidates found this question on resolving forces rather more challenging than the first two questions. Most candidates resolved horizontally and vertically rather than parallel and perpendicular to X, although the with the latter method there was no need to solve simultaneous equations since the value of the tension could be obtained directly. Those who used this latter method usually gained full credit. Of those who used the former method, most candidates resolved correctly in the vertical direction. However, some candidates only included one tension in their horizontal equation and many others thought that there were two different tensions, not recognising it was a single string.

Question 4

(a) This question also proved to be rather demanding despite the given answer and there was a wide variety of approaches, all involving Newton's second law. Most candidates found two equations for either $\frac{P}{v}$ or for F in terms of a and $\frac{1}{2}a$. These were usually correct, although there were often sign errors seen. Some candidates then used a calculator to solve the equations, which was not satisfactory as it was required to 'show' that the power was 200 kW. One of the most popular methods was to express P in terms of a for both equations and equate the two expressions. Once candidates had solved for a, they then had to find P. Some did not have an explicit equation for P but instead simply wrote $\frac{P}{20} - 6000 = 15000a$ or similar, followed by $P = 200\,\text{kW}$ without first showing a substitution of the value of $a = \frac{4}{15}$. Omitting this substitution meant they could not be awarded the final mark. Some candidates tried to use the given value of P to find a but most of these only had one equation, usually $\frac{200\,000}{20} - 6000 = 15000a$, and unless they had a second equation they could only gain a maximum of two marks.

(b) Candidates were slightly more successful with this part, although despite the question asking for the steady speed that the lorry could maintain, a significant number included a non-zero acceleration in their work.

Question 5

(a) In this question on calculus, this first part was almost always answered correctly.

(b) This part was also very well answered. Most candidates integrated correctly and used the correct limits. Some made extra work for themselves by finding the constant of integration rather than simply using a definite integral. A few found both areas correctly but forgot to state that one was a tenth of the other. A few used a calculator to integrate which was not a satisfactory method for a 'show' question.

(c) There were many correct responses to this part, but there were also quite a few different errors which resulted in candidates not gaining full marks. A few candidates made an error in differentiation, some differentiated twice and stated that the maximum acceleration was 0.4 ms^{-2}, since this was the value of the second differential. Some only worked out the value of 0.4 without also finding the maximum acceleration of 0.3 ms^{-2} in the first 9 seconds, and others, although they found this value, did not clearly state which was the maximum.

Question 6

(a) The first two parts of this question were straightforward problems involving constant acceleration and both parts were very well done, although some candidates thought that the constant speed section ended at $t = 25$ rather than $t = 30$. Such candidates usually still had the correct time of 10 seconds for the deceleration.

(b) This part was usually fully correct apart from those candidates who had the error mentioned in **part (a)**. These could still get a mark for the correct method. Some candidates who had made this error in **part (a)** gained full marks in **part (b)** as they used equations for constant acceleration to find the total distance travelled, rather than using their diagram from **part (a)**.

(c) In this part, candidates had to use Newton's second law to find the value of the mass. This question was found to be quite challenging, with relatively few fully correct responses. Of those who made some progress, some omitted one of the terms or included an extra term. Rather more had an incorrect sign in their equation, often getting a final answer of 0.98 kg.

(d) There were very few correct responses to this part, again involving Newton's second law. Many candidates had no idea of how to proceed. Some used the wrong value for deceleration, usually 0.2 or g, and others included the mass of the elevator. Some had an equation with a sign error, but which was otherwise correct.

Question 7

(a) This question involved using a work-energy and was found to be somewhat challenging. Most candidates correctly found the loss in potential energy and some then went on to correctly find the velocity. However, many candidates had an incorrect sign in their work-energy equation or omitted the work done by the child, so had a two-term equation only. A few candidates tried to use an acceleration method which was not correct, since the work done was not stated to be constant and the child was moving on a curve.

(b) This part, which involved using an energy method to find the coefficient of friction, was again found to be challenging. Many candidates correctly found the loss in potential energy and the normal reaction force. However, often the equation for the work done by the friction force was not correct. Many candidates either had an incorrect sign, or more often omitted the distance term in the friction force and so had an equation which was not dimensionally correct, since one the terms in the work-energy equation was just a force. Candidates usually realised that they had to use the relationship $F = \mu R$ and some correctly found the value of the coefficient of friction, but some did not notice that the answer had to be given as a fraction in its simplest form. Some candidates gave, for example, the normal reaction force as $25g\cos\alpha$, but then did not evaluate this expression. Candidates should be aware that marks are only awarded once a value is substituted for α, or when $\cos\alpha$ is evaluated (in this case as 0.96).

MATHEMATICS

Paper 9709/51
Probability & Statistics 1 (51)

Key messages

Candidates need to be aware that workings and explanations are required to support their answers. It is especially important to include all the required steps when the proof of a given result is called for, including the mathematical operations. Good solutions were characterised by clear communication, particularly when combining different results or scenarios.

Where a diagram is required, it should be clear, accurate and appropriately labelled.

Candidates should only state non-exact answers correct to 3sf; exact answers should be stated exactly. To justify a final answer correct to 3sf, working values correct to at least 4sf should be used in the calculations throughout. There is no requirement for fractions to be converted to decimals.

General comments

Most candidates used the response space effectively. Where there is more than one attempt at a question, candidates should ensure that they clearly identify which one they intend to present for marking. When extra space is required, candidates should use the additional page in the first instance.

The use of helpful diagrams, sketches and tables were frequently seen in good solutions. They often supported and efficiently organised the explanations. Many candidates were able to tackle the earlier parts of **Questions 1, 2, 3** and **4**. Frequently, the latter parts of **Questions 1, 2, 3** and **7** seemed challenging for many.

Comments on specific questions

Question 1

Many candidates found this question challenging and were unsure how to deal with the summary statistics given in coded form. There were a significant number of scripts with no response to this question.

(a) Good solutions often found the mean of $x - q$ as 14 first and subtracted its square from $\frac{14235}{50}$ before square rooting to find the standard deviation. The correct answer of 9.42 often followed. A few answers were given only to 2sf. Weaker solutions simply found the square root of $\frac{14235}{50}$ or tried to expand the brackets, whilst others squared the 700 but not the 50.

(b) Successful candidates understood that $\sum q = 50q$ and formed the correct equation with $\sum x$ and $\sum (x-q)$, whilst others found the mean of x and subtracted the mean of $(x-q)$ from it. Many did not attempt this part or simply found $\frac{2865}{50}$ or 2165, not understanding how to use the coding.

Question 2

Many candidates were able to tackle **part (a),** but many found the restrictions in **part (b)** more difficult.

(a) Most candidates identified correctly that combinations were required to find the number of ways that the committee was chosen. The correct combinations of men, ${}^{6}C_{3}$, and women, ${}^{8}C_{3}$, were frequently seen. Many successfully found their product and gave the right answer. A significant number of candidates found their sum.

(b) Stronger candidates often gave an indication that the number of brothers required were 0, 1 or 2 and found the number of ways of selecting the remaining candidates from 11 people in each case. This situation, which requires the multiplication of combinations and then the addition of scenarios, is quite common. A few who adopted this method selected the remaining committee members from 8. Some omitted one of the scenarios (most often 0 brothers) when adding to get the required answer. A few successfully listed all the 12 different scenarios of men and women for each number of brothers and found the associated combinations. Many of those who attempted this method did not get all the scenarios and would have been well advised to list their options in a logical manner. Some used the efficient method of subtracting the number of ways that all 3 brothers could be selected from the total number of ways the committee could be selected. Most of these found the total number of ways of selecting the committee, ${}^{14}C_{6}$, but were unsure what to subtract for 3 brothers.

Question 3

There were a lot of good solutions to **parts (a)** and **(b)** which demonstrated understanding of arrangements with repeated Os and Cs. Many candidates found **part (c)** less accessible and were unable to deal with the conditional probability.

(a) The number of arrangements of 8 letters where 3 are Os and 2 are Cs is a fairly standard application involving factorials and was completed successfully by many. Some solutions indicated that candidates thought that the Os and Cs were distinguishable and just gave $8!$.

(b) Most candidates realised that they had only 6 letters to arrange. Stronger solutions were often accompanied by a simple diagram which showed the number of spaces to be filled and the letters that were left to fill them. Some omitted the denominator of $2! \times 2!$ to deal with the repetition of the remaining Os and Cs.

(c) A significant number of candidates found this part more demanding. Good candidates realised that this was an application of conditional probability where there were 5 items to arrange once the Os and Cs had been grouped together and 7 items to arrange (3 of which were Os) where the 2 Cs were next to each other. Weaker solutions simply found the number of arrangements with 3 Os together and 2 Cs together and divided the result by 3360, their answer to **part (a)**.

Question 4

The vast majority of candidates were able to use the Normal standardisation formula correctly. Most candidates correctly identified that, as time is a continuous variable, no continuity correction was necessary.

(a) The best solutions were often accompanied by a simple diagram to inform the method. In a few instances a continuity correction was applied, or the variance was used. Candidates must appreciate the need to use the full value obtained from the Normal distribution table to obtain an accurate probability. Some stopped after finding the probability; candidates would be well advised to ensure that they read through each question carefully to ensure that they fulfil all its requirements. Some better responses showed calculations of the number of students by multiplying by 250 and appreciated the need to give their answer as an integer without extra solutions or any reference to rounding.

(b) The finding of μ and σ from given information is a familiar application of the Normal distribution. Strong candidates often provided a supporting diagram identifying the probabilities given in the question. They went on to give the correct *z*-values corresponding to those probabilities in 2 standardisation formulae. Information in their diagram allowed them to see that both *z*-values should be negative. Most candidates who obtained the formulae were able to solve their system of equations to provide answers for both μ and σ. Some candidates prematurely rounded their *z* values leading to inaccurate solutions. Weaker solutions used the given probabilities as *z* values in the formulae.

Question 5

Candidates would be well advised to consider the nature of the data groups provided in the question with relation to the boundaries for each class before drawing their diagram. A large proportion did not fulfil the requirements of the question and drew a bar chart using the original data.

(a) Good solutions stated the frequency densities (using the correct boundaries for each class) before drawing the graph. This assisted the correct selection of a suitable scale. Scales must be selected to permit all the data to be represented, the population scale ending at 4800 was often seen. The data allowed a simple scale of 2 cm representing 1000 on the population axis and 2 cm representing 0.01 on the frequency density axis. Many candidates used the correct class intervals, but a significant number were inaccurate when marking them along the axis. Those who calculated the frequency densities correctly were often unable to plot 0.0625 in the correct place. The careful use of a ruler is essential to ensure that lines drawn are on the grid lines when necessary and along part squares where necessary. Graphs should always be drawn with a sharp pencil to make this clear. Where the bars are coloured in, it is not always possible to check if the class boundaries are correct. If an error occurs, careful erasing should ensure that the necessary correction can be made. Many candidates left gaps between the bars, not realising the significance of the words 'to the nearest 100' or drew a bar chart with the original data. The frequency density axis was often labelled correctly, but in many cases candidates did not reference the data table to select the label of 'village population' for the horizontal axis.

(b) Many candidates were able to select the correct interval for the median. A few weaker solutions gave 1300 – 2000, which was the middle interval, or gave 75, which was half of the number of villages.

(c) Good solutions identified the classes in which the upper and lower quartiles lay and a few of these appreciated that they needed to subtract the lowest value for the lower quartile from the highest value for the upper quartile. The calculation 3200 – 1300 was very often seen, not allowing for the correct boundaries in which the quartiles lay. Weaker solutions used the values needed to find the quartiles and calculated 112.5 – 37.5.

Question 6

Good solutions were not often seen in this question. Those who multiplied the correct probabilities for the number of 2s often neglected the number of ways this could be achieved.

(a) Good solutions were often accompanied by a list showing all the possible outcomes which ensured that the product of the probabilities was multiplied by 4 or ${}^{4}C_{3}$. Some good candidates did not appreciate the rigour required in a 'show that' question and needed to include the multiplication of probabilities to justify their answer. Weaker solutions, where the score on only 3 of the dice was considered to manufacture the given answer, were often seen.

(b) Stronger candidates were able to apply the method suggested in **part (a)** to find both missing values in the table. Many were able to score a mark by realising that the two missing values should sum to $\frac{81}{128}$, as the sum of the probabilities must be 1. In a few cases the probabilities found were not placed in the table; candidates would be well advised to read the requirements of the question carefully.

(c) Many candidates were able to use the correct method for calculating the expectation from their probability distribution table. Candidates are reminded to show where their values come from in order to score the method marks, even if they are using incorrect values from **part (b)**.

(d) The strongest candidates used the appropriate probability of scoring at least two 2s of $\frac{67}{256}$ to calculate the mean and variance of the approximating Normal distribution. Some candidates used the probability of scoring 2; reading the question carefully is always important. The best solutions included a helpful diagram and an appreciation that a continuity correction was necessary since the original data was discrete. Candidates are reminded of the need to work accurately, as premature rounding denied some the final mark. Fully correct solutions were rarely seen with a significant number of candidates not attempting this part or attempting to use the Binomial distribution in some form.

Question 7

Many candidates were uncertain about using the Geometric distribution and probability theory in this question, but many were able to use the Binomial distribution in **part (b)**.

(a) The approach most often seen was the addition of the probabilities of obtaining the elephant for the first time on the 1st, 2nd, 3rd, 4th or 5th day. Those who used the Geometric distribution often produced correct and efficient solutions. Occasionally, $1 – 0.8^6$ was seen. Many candidates found this part challenging, with some attempting to use the Binomial distribution. Solutions only finding the probability of getting an elephant for the first time on the 5th day were often seen.

(b) This part was completed successfully by a good number of candidates. The efficient approach of summing the probabilities of 0, 1 and 2 leopards and then subtracting from 1 was used by the majority of candidates. Those who attempted to sum the probabilities of 3 – 12 leopards were rarely successful due to errors in accuracy or omitting one or more of the outcomes. In a few instances the binomial coefficients were incorrect or missing. Candidates would be well advised of the need to work to at least 4sf to justify an answer to 3sf, as highlighted in the Key messages. Some gave a final answer to 2sf only.

(c) Correct solutions to this part were rarely seen. Many candidates realised that to get one of each animal 0.2^5 had to be calculated, but few understood that the arrangements of the animals had to be considered as they were all different. The product of 0.2^5 and $5!$ was rarely seen. Candidates who considered the number of ways of selecting the 1st, 2nd, 3rd, 4th and 5th animals, then dividing by the total number of ways of arranging the animals, were more likely to include the $5!$. This approach was not often used.

MATHEMATICS

Paper 9709/52
Probability & Statistics 1 (52)

Key messages

Candidates should be aware of the need to communicate their method clearly. Simply stating values often does not provide sufficient evidence of the calculation undertaken, especially if there are errors earlier in the solution. The use of algebra to communicate processes is anticipated at this level and enables candidates to review their method effectively and is an essential tool when showing given statements are true. When errors are corrected, candidates would be well advised to cross through and replace the term. It is extremely difficult to accurately interpret terms that are overwritten.

Candidates should state only non-exact answers to 3 significant figures, exact answers should be stated exactly. In particular there should be a clear understanding of how significant figures work for decimal values less than 1. It is important that candidates realise the need to work to at least 4 significant figures throughout to justify a 3 significant figures value. Many candidates rounded prematurely in normal approximation questions which produced inaccurate values from the tables and lost accuracy in their solutions. It is an inefficient use of time to convert an exact fractional value to an inexact decimal equivalent, there is no requirement for probabilities to be stated as a decimal.

The interpretation of success criteria is an essential skill for this component. Candidates would be well advised to include this within their preparation.

General comments

Although many well-structured responses were seen, some candidates made it difficult to follow their thinking within their solution by not using the response space in a clear manner. The best solutions often included some simple notation to clarify the process that was being used.

The use of simple sketches and diagrams can help to clarify both context and information provided. These were often seen in successful solutions. Candidates should be aware that a ruler should be used to construct box-and-whisker plots and that the scale should enable to accurate plotting of the five key-values to be achieved.

Sufficient time seems to have been available for candidates to complete all the work they were able to, although some candidates may not have managed their time effectively. The vast majority of candidates were well prepared, however candidates found it more challenging when more than one technique was required within a solution. Many good solutions were seen for **Questions 1** and **3**. The context in **Questions 2, 5** and **6** was found to be challenging for many.

Comments on specific questions

Question 1

Many good solutions to this question were seen.

(a) A probability distribution table was present in nearly all solutions. A large number of candidates formed an equation for the probabilities and calculated k before entering values into the table. Better answers did not find a numerical probability at this stage, but left the probabilities in terms of k. Weaker solutions included probabilities in the table which did not sum to 1, or were negative. Even though the value for the random variable X were stated in the question, some additional outcomes with probabilities were seen.

(b) Most candidates who had formed a probability distribution table in **part (a)** made good progress here. Many solutions included the full, unsimplified calculations for both $\mathrm{E}(X)$ and $\mathrm{Var}(X)$, which is good practice. The efficient use of a calculator allows these to be evaluated without further simplification while clearly communicating the method. A small number of solutions failed to use $(\mathrm{E}(X))^2$ in the variance calculation. As the question required two values to be found, candidates should be aware of the expectation that appropriate identification is provided.

Candidates who had not found a value for k in **part (a)** often solved here, but would not gain any credit, and complete the question accurately.

Question 2

A tree diagram was present in many solutions. This clarified the probability information significantly and, with effective labelling, enabled accurate interpretation of the success criteria throughout the question.

(a) The appropriate probability calculation was seen in most solutions. The most common error was not recognising that an exact value had been calculated and providing only a three significant figure answer. Candidates should be aware that the instruction on the front of the paper is that 'non-exact' answers are rounded, so exact values should be stated.

(b) Solutions which included a tree diagram were often successful. Most candidates recognised that they needed to consider only Sunday, Monday and Tuesday in determining the required probability. Those who included Wednesday often used 0.8 and 0.2 as the probabilities, rather than 0.7 and 0.3, since it had rained on Tuesday. A small number of solutions ignored the context and simply stated a probability of 0.2 that it rained after a day without rain.

(c) A significant number of candidates made little or no attempt at this part. The most successful solutions clearly identified the possible scenarios which fulfilled the criteria and then calculated the probability of each. Many candidates simply stated calculations and did not clearly communicate the logic that was being used. There were an unexpectedly high number of arithmetical errors in the evaluation of the expressions stated. As the probability was an exact value, candidates should not round to 3 significant figures.

Question 3

Almost all candidates were able to interpret the back-to-back stem-and-leaf accurately.

(a) The value for the median was found accurately by most candidates. As more than one item was demanded by the question, candidates should be aware of the need to identify each answer appropriately. The calculation of the interquartile range was less consistent, especially in determining the value of the upper quartile. Most candidates used the anticipated method of determining the middle value between the maximum or minimum value and the median to find the quartiles. There is an expectation that a calculation will be present or clearly implied for the interquartile range. A small number of candidates did not use the information from the stem-and-leaf key to scale their answers.

(b) The majority of candidates used an appropriate scale for their box-and-whisker plots. This enabled the key values to be plotted accurately, with the majority of values on grid lines. Many plots were drawn without a ruler, which is not appropriate at this level as an accurate representation of the data is expected. A small number of candidates failed to label the separate companies, so their comparison was unclear. Candidates should be aware that a linear scale is required and that it should be labelled with both the variable and units (e.g. salary in $). Several candidates presented solutions with the plots of company *A* and *B* combined, which gained no credit.

(c) Many general, theoretical comments were noted. Candidates should be aware that comments need to be within the context of the question, and specific to the data presented. Reference to the n extreme value of $3090 in company *B* was expected, or to be clearly implied in the comment. There appeared to be some confusion about whether the inclusion of the extreme value in the central tendency was appropriate or not.

Question 4

Many candidates found this probability question challenging. **Part (d)** was omitted by a significant number of candidates but was often completed successfully by those who attempted it, since much of the process was fairly standard.

(a) Almost all candidates recognised the geometric approximation was appropriate for the context, and stated the anticipated calculation, which was evaluated accurately. As this produced an exact answer, no rounding should have been undertaken for the final answer.

(b) Many good solutions were noted for this part. These often used the less efficient process of adding the probabilities of obtaining a 2 in each of the possible acceptable scenarios. The more efficient process using $1-0.8^5$ was used effectively from more confident candidates. As has been highlighted in previous reports, misinterpreting the success criteria is a very common error, and many solutions included the 6^{th} spin as well.

(c) This conditional probability was found challenging by many candidates. A surprisingly high number of candidates omitted this part entirely. The most successful solutions used an outcome table, which identified all the possible scores that could be obtained. The conditional probability could then be stated directly by identifying the appropriate values. However, candidates were usually more successful when they used the outcome table to support the probabilities to substitute into the appropriate conditional probability fraction. Weaker solutions often used a listing approach, with outcomes omitted, or tried to use logical reasoning to find the required probabilities.

(d) This part was also omitted by a significant number of candidates. Good solutions were noted frequently when attempted. Many candidates found the context challenging, although when an outcome table had been used in **part (c)**, the required probability could be identified. The mathematical process was a fairly standard binomial approximation context. As in **part (b)**, misinterpreting the success criteria was a common error, with three occasions being included. Poor arithmetical accuracy was seen, not always linked with premature approximation as can be expected. Candidates should be aware that the efficient use of the calculator should enable them to evaluate the entire unsimplified expression without any intermediate values being stated, which can avoid both premature approximation and rekeying errors.

Question 5

This was a relatively standard normal approximation question. The number of candidates who did not attempt **part (c)** was higher than anticipated.

(a) Almost all candidates used the normal standardisation formula correctly at least once. The best solutions often had a sketch of the normal distribution curve to help identify the required probability area. Very few candidates used a continuity correction, which was not required as the data is continuous. A common misinterpretation was that the required probability area was symmetrical and, although the correct standardisation formulas were stated, the second evaluation was assumed and not calculated.

Many solutions finished when the probability area had been calculated, with the final process required to find the expected number of birds that fulfilled the criteria not attempted. Candidates are well advised to read the question again once they reach their final answer to check that it is both reasonable and does answer the question. Because of the context, the final answer needed to be an integer value and there should be no indication of rounding as the value chosen is a decision from the candidate. Candidates should also be aware that evidence of a probability to at least four significant figures is required to justify their decision.

(b) A fairly standard normal approximation technique was required for this question. Good solutions often used a sketch of the normal distribution curve to identify the magnitude of the anticipated *z*-value. Candidates who formed an appropriate equation using the normal standardisation formula were frequently successful in determining the value of σ. Many candidates found a probability value rather than a *z*-value from the information, so were unable to form an appropriate equation.

Candidates should be aware that their supporting work needs to be consistent throughout the solution. A number of candidates realised that the standard deviation could not be negative so stated a positive value for their final answer, but did not amend their initial error of using the incorrect *z*-value.

(c) Again, even though this question was not attempted by many candidates, several good attempts at this standard normal approximation question were seen. The best solutions had calculations of the mean and variance initially and then a substitution of these values in the normal approximation formula. Most candidates recognised that the data was discrete and so required a continuity correction, although a few used the upper rather than the lower bound. The most common error was to use the incorrect probability area, which may have been avoided if a simple sketch of the normal curve was used to clarify the success criteria.

Question 6

This permutations and combinations question was found challenging by many, but candidates who listed logically possible scenarios often achieved good solutions. Candidates should be aware that how the question is presented provides some additional guidance for their solution. As additional information was presented before **part (b)**, this is additional context for the remainder of the question and may be required not only in **part (b)** but also in **part (c)**.

(a) Candidates who listed the three possible scenarios that fulfilled the criteria were frequently successful. The best solutions included the unsimplified calculations involving the combinations linked with each scenario. A misconception was that as the number of swimmers was fixed at 1, which would have no effect on the ways the teams could be selected, so ${}^{6}C_{1}$ was omitted from each product. Weaker solutions summed the combinations to find the number of outcomes for each scenario.

(b) Candidates who used a simple diagram to clarify the requirements given in the question were often able to identify that not only could the team be arranged within their groups, but that the groups themselves could be arranged in different orders. Common errors were either not to arrange the groups, with an answer of 288, or not to arrange the people within the groups, with an answer of 6. A number of candidates assumed that they needed to select the identified people from the original group, and so increased the complexity of the question. As the additional information was given before the question was started, this is an indication that the previous conditions would no longer be applied.

(c) Many candidates found this part challenging, and the question was not attempted by a surprising number. Two main approaches were used to answer the question. The most successful was to consider how the people who were not cyclists were arranged and then how the cyclists could be placed. Good solutions often used a simple diagram to clarify the context. A common less successful method included deducting the number of arrangements where the cyclists were standing together from the total number of arrangements for the team. Most candidates who used this approach only attempted to deduct the cases where all three cyclists were together, and failed to consider that just two cyclists could be together with the final cyclist being placed elsewhere. Again, more successful solutions used simple diagrams to clarify the criteria and more explanation was included in the work to inform what was being considered.

MATHEMATICS

Paper 9709/53
Probability & Statistics 1 (53)

Key messages

More marks were awarded this session due to a larger number of candidates showing their method. However, a minority still need to be reminded that they must show their working and justify their answers. In **Question 2** we needed to see the standardisation formula with the correct values substituted, in **Question 3** we needed to see how they derived their values for a and k, in **Question 4a** we needed to see the subtraction of the lower quartile from the upper quartile and in **Question 6b** we needed to see the binomial terms in full.

General comments

A surprising number of candidates struggled with **Question 1** and did not seem comfortable with the geometric distribution. Conversely, **Question 7**, which required the use of Permutations and Combinations and is a topic which normally causes problems, was confidently dealt with by most candidates. A significant number of candidates struggled with the algebra in **Question 3**, especially expanding the brackets.

Comments on specific questions

Question 1

(a) This proved to be a challenging starter question for the many candidates who did not recognise a geometric distribution and remember that $\mathrm{E}(X) = \frac{1}{p}$. Some formed a probability distribution table and incorrectly used the formula $\mathrm{E}(X) = \sum x \cdot p(x)$, not appreciating the significance of the word 'State', i.e., that no working should be needed. Many worked with $\frac{1}{2}$ as the probability while the most common error was to give $\frac{1}{4}$ as the final answer.

(b) This question was answered more confidently, although a significant number of candidates confused three decimal places with three significant figures and dropped the last digit in their final answer. Those who worked with $\frac{1}{2}$ as the probability in **part (a)** often continued to do so in this part as well.

(c) Strong candidates knew to raise the probability of not getting a pair of heads to the power of 6 and subtract the result from 1. Others used the longer method of summing the probabilities of getting a pair of heads in 1, 2, 3, 4, 5 or 6 throws. Both methods were equally valid, but the second method was more prone to arithmetic errors. As in the previous parts of the question, a significant number worked with a probability of $\frac{1}{2}$ instead of $\frac{1}{4}$.

Question 2

This question was answered well with most candidates recognising what was required from a normal approximation to the binomial. However, the words 'between 36 and 54 inclusive' seem to have been disregarded or misunderstood by a significant number of candidates, with many of those who did remember the continuity correction using it in the wrong direction. Almost all candidates found the correct mean and variance and used them correctly in at least one standardisation expression. Evidence of standardisation was required here and candidates who used their calculators and went straight to the *z*-values were penalised. Most found the correct area between their two *z*-values, often using a sketch to help.

Question 3

(a) The question prompted candidates to form their first equation from the given information, $\mathrm{P}(X=4)=3\mathrm{P}(X=2)$. Most correctly stated that $4k(4+a)=3\times 2k(2+a)$ and stronger candidates quickly cancelled the k on both sides of the equation and worked out that $a=2$. Others expanded the brackets and simplified until reaching the equation $4k=2ak$. Most then cancelled the k and arrived at the correct value for 'a', but in several cases they stalled at this late stage and never found the value of 'a'. A disappointing number made careless algebraic mistakes while expanding the brackets.

Many candidates never formed a second equation by summing the probabilities to 1. Some substituted $a=2$ back into their original equation, giving $24k=24k$, and decided that $k=1$. Those who did remember the more usual way of dealing with a probability function generally obtained the correct value for 'k'.

(b) This was well answered with almost all knowing how to form a probability distribution. Despite the instruction to give the probabilities as 'numerical fractions', a significant number gave these algebraically.

(c) Most candidates were familiar with the formula for calculating the variance from a probability distribution and knew to show the sum of their squared x-values multiplied by their probability before they subtracted 3.2^2. Some ignored the fact that they were given the value of $\mathrm{E}(X)$ and calculated it for themselves. Only a few still did not realise that they need to show their numerical substitution into the variance formula if they are to be awarded the marks.

Question 4

(a) Finding the median and interquartile range from a back-to-back stem-and-leaf diagram was a familiar task for most and accurately answered. A few were confused by the backward left hand Cheetahs figures and gave the upper quartile or 15$^{\text{th}}$ item of data as 101 rather than 106.

(b) When asked to make comparisons between sets of data, candidates should be encouraged to write answers in context and with an appropriate interpretation. This question challenged even the strongest candidates, with many simply comparing numerical values. When comparing the medians, we needed to read that generally the Cheetahs were faster, or completed the race in a shorter time, or that generally the Panthers were slower or took longer to complete the race. When comparing the interquartile ranges, we needed to read that the Panthers' times were more consistent or less spread out, or that the Cheetahs' times were more varied or more spread out.

Any comparisons of particular values, e.g., medians, averages, ranges or interquartile ranges, were not awarded marks.

(c) This question was answered well by most candidates. Strong candidates found the total time of the 20 including Kenny by multiplying 99 by 20 and then subtracted the total of the 19 that feature in the stem-and-leaf diagram. Some chose to find the average of the 19 which was 98 and then added 20 × 1, where 1 is the difference between the average of 19 and the average of 20. Apart from some who made arithmetic errors there were very few incorrect responses.

Question 5

(a) The most successful approach to finding the probabilities of A and B seemed to be using two grids showing the different outcomes. Without this, even the initial step in finding the total number of outcomes caused problems. Notation was generally very good although use of the intersection caused the most problems, with alternatives seen including: $\mathrm{P}(A$ and $B)$, $\mathrm{P}(A+B)$, or simply $\mathrm{P}(AB)$.

The vast majority chose to use the independence condition $P(A \cap B) = P(A) \times P(B)$. Even weaker responses showed an attempt to multiply the two probabilities, demonstrating an awareness that this was linked with independent events.

(b) Strong candidates who had correctly evaluated the required probabilities in the previous part knew to divide $P(A \cap B')$ by $P(A')$ and usually obtained the correct final answer. Most explained that they had subtracted $P(A)$ from 1 to find $P(A')$ and some even explained that they had subtracted $P(A \cap B)$ from $P(B)$ to find $P(B \cap A')$. Weaker candidates often knew to divide by $P(A')$ but were less successful with the numerator, often assuming independence and multiplying $P(B)$ by $P(A')$ or just using $P(B)$.

The few who used the second method in the mark scheme and worked directly from the outcome tables were generally successful.

Question 6

(a) It was imperative that candidates used the critical value of 1.282 for this part; use of 1.2815, 1.281 or 1.28 was not accepted. Most candidates knew to equate a standardised expression using 16, 28 and sigma to a *z*-value with only a small number using the tables backwards or equating to 0.1 or 0.9.

(b) The majority recognised this as a binomial question and only a few incorrectly tried to work with the normal distribution. Understanding the words 'more than 2' was an issue for some, with a number omitting the probability of '2 days' from their calculation. A few found the probability of 0, 1 or 2 days and then did not go on to subtract from 1 and a few others only gave the answer to two significant figures.

It was pleasing to see that most candidates realised we need to see the method with all the binomial terms.

(c) This proved to be a challenging question and only the candidates with a firm grasp of the normal distribution were confident enough to jump directly to the correct *z*-values of +/– 1.3. Many others obtained the correct *z*-values after standardising using 28 and 9.36 but, unless they realised that the expressions easily simplified to +/–1.3, they often obtained inaccurate results. A common misconception was to treat +/– 1.3/9.36 as the required *z*-values.

Most candidates did obtain a probability, but many forgot to do the final part and answer the question about how many days they would expect the mass of grapes to be within 1.3 standard deviations of the mean. We insisted on an integer value for the final answer and that they multiplied 365 by a 4-figure probability.

Question 7

(a) The two methods in the mark scheme were seen and both were generally performed well.

In Method 1, they calculated the total number of ways of arranging the 10 letters, i.e. $\frac{10!}{2!4!}$, and subtracted the number of ways the letters could be arranged with the two Cs together, i.e. $\frac{9!}{4!}$.

In Method 2, they calculated the number of ways the 8 letters apart from the Cs could be arranged, i.e. $\frac{8!}{4!}$, and then multiplied by the number of ways the Cs could be inserted into the arrangement without being together (${}^9C_2$).

(b) This part of the question was more challenging. Strong candidates quickly realised that there were $\frac{6!}{2!}$ ways of arranging the 6 remaining letters, after disregarding the As at the beginning and the end and the Cs. They then multiplied by 4 as there are four ways of positioning the block of 5 letters containing the two Cs and three other letters.

A slightly more complicated way was to see that there are $\frac{{}^6P_3}{2!}$ ways of arranging the 3 letters between the two Cs, $3!$ ways of arranging the other three letters and four ways of positioning the block of five letters. Candidates who thought about the problem in this way were more prone to error.

A significant number of candidates unnecessarily complicated the problem by thinking that they had to consider how many of the remaining two As were between the Cs. They made three calculations, considering 0, 1 or 2 As between the Cs and then added them. An impressive number of candidates used this complicated approach and arrived at the correct final answer, but many others went wrong.

Only a few made a start at the highly inefficient method of finding the number of ways with an A at the beginning and the end, namely $\frac{8!}{2!2!}$, and then subtracting the number of ways without three letters between the Cs. This would involve considering the six scenarios of 0, 1, 2, 4, 5 or 6 letters between the Cs and most gave up very quickly.

(c) This question was very well answered by many candidates. Very few used Method 1 where they only considered the number of As. With this method, they can disregard one of the Cs and select from 5 letters (C, S, B, L, N) to go with each of 2, 3 or 4 As. They then total the number of ways for the 3 scenarios.

With Method 2, most candidates successfully identified the six scenarios with 2, 3 or 4 As and 0 or 1 C and only a few omitted to show that essential stage of their working. Most appreciated that once they had determined how many Cs and As were in a scenario that only left four other letters (S, B, L, N) from which the remaining letters should be selected. Having found the number of ways for each of the six scenarios they then totalled them.

The most common error was to think that each number of As or Cs can be selected in multiple ways. For example: to find the number of ways of AAAC which should be ${}^4C_1$, i.e. the number of ways of selecting the remaining letter, they multiplied ${}^4C_3$ by ${}^2C_1$ by ${}^4C_1$.

MATHEMATICS

Paper 9709/61
Probability & Statistics 2 (61)

Key messages

- In all questions, sufficient method must be shown to justify answers.
- It is important that candidates read the question carefully and refer back to it when they have completed the question to ensure they have answered it in full.
- Candidates are strongly advised to carry out all working to at least four significant figures if a final answer is required to three significant figures.
- For answers that are required 'in context', quoting general textbook statements will not be sufficient.
- All working should be done in the correct question space of the answer booklet. If answers need to be continued on the Additional page, it must be clearly labelled with the correct question number.
- Candidates should make corrections by crossing through and replacing the work, not by over-writing their answer.
- Only one solution should be offered.

General comments

Candidates did not always seem fully prepared for the demands of this paper. Questions where candidates performed well were **Questions 1, 3, 7(a)** and **7(e),** and questions which candidates found more demanding were **2(a)**, **2(b)**, **4(b)**, **4(c)**, and **7(d)**. There were a few places on the paper where it appeared that candidates had not read the question carefully, namely **Question 3** and **Question 7(c)**.

Candidates must note that the conclusion to a hypothesis test must be written in context and with a level of uncertainty in the language used.

Comments on specific questions follow which identify common errors, though it should be noted that there were many good and fully correct solutions seen as well.

Comments on specific questions

Question 1

(a) In general, this part was well attempted. The approximating distribution required was a Poisson distribution $\text{Po}(3.4163)$. Many candidates calculated the correct proportion of adults but did not realise that a Poisson distribution was required. Some candidates used a binomial distribution or attempted a normal distribution, and of those who did attempt a Poisson distribution, accuracy was often lost (caused by premature rounding of 3.4163). It should be noted that when calculating a probability such as this sufficient method must be shown, so the Poisson expression needs to be clearly written in full.

(b) To justify an approximation, it is important that the context of the question is used. As highlighted in the Key messages, merely quoting from a textbook ($n > 50$, $np < 5$) is not sufficient. The values of n and np in the given situation need to be clearly stated to demonstrate that they do fulfil the requirements.

Question 2

(a) Some candidates successfully calculated the area under $f(x)$ and showed it was equal to 1, as required for a probability density function. However, very few candidates stated $f(x) \geqslant 0$, which is also a requirement.

(b) Many candidates omitted this question, and of those that did make an attempt, many chose to use a set method by integration rather than to use the formula for the area of a semi-circle and equate it to 1. It is important that candidates have an understanding of the methods they are using, and that sometimes use of integration is not always the best method to find an area. In general, there were few good attempts seen for this part.

(c) (i) A few candidates were able to give an acceptable explanation here. Candidates who discussed the skewed nature of the curve were not always successful in their reasoning. Mentioning area to the left and right of 15 was generally a more successful approach.

(ii) Integration attempts on this part were not always successful. Some candidates integrated $\mathrm{f}(x)$ rather than $x\mathrm{f}(x)$ and some confused the mean and median.

Question 3

Many candidates made a reasonable attempt at this question. There were some good solutions but equally some that did not carry out all the required steps fully. Some candidates omitted or gave incorrect hypotheses. Comparisons were not always valid or clearly stated, and very few conclusions were written with the required context and with a level of uncertainty in the language used. As highlighted in the Key messages, many candidates did not state a necessary assumption, highlighting the importance of reading the question carefully and checking back when finished to ensure that the question has been fully answered.

Question 4

(a) Candidates who successfully set up the correct equation were usually successful in finding n. Errors setting up the equation included using an incorrect z value, and more commonly, to omit the factor of 2.

(b) This was well attempted by only a few candidates. Many candidates stated only the first 3 months of the year had been chosen, but this was not sufficient to explain why this made the sample unsuitable. Further comments that this sample was not typical of the whole year, possibly including comments about weather or similar, needed to be made. Some candidates made comments about the sample size which were not accepted.

(c) Only a few candidates realised what was needed here, with a large number of candidates giving no response at all. Some candidates stated part of the expression but there were very few fully correct answers.

Question 5

(a) Many candidates were unsure of how to calculate the variance. Of those that did, a common error was to use 20^2 rather than 20.

(b) Some candidates successfully found the correct mean and variance and standardised to find the correct probability area. Errors were seen in both finding the variance and the correct probability area (> 0.5 rather than < 0.5), but in general reasonable attempts were made.

Question 6

This question received a wide range of responses, from totally correct to little or no response. Setting up the initial equation which required the correct formula for the unbiased estimate of the variance was reasonably well attempted, but many errors were made in solving this equation. A common algebraic error was expanding $(10 + a)^2$ incorrectly as $100 + a^2$.

Question 7

(a) There were many correct responses, but some candidates incorrectly used p or $\bar{x}$ rather than λ (or μ). On occasions this was omitted completely, with candidates stating $H_0 = 1.9$ and $H_1 < 1.9$ or $H_0 = 7.6$ and $H_1 < 7.6$.

(b) Candidates were required to evaluate $P(X \leqslant 2)$ and $P(X \leqslant 3)$ in order to find that the critical region was $X \leqslant 2$. Some candidates did not know how to approach the question, others used an incorrect value for λ (often 1.9) or merely found individual point probabilities. Some candidates correctly identified the critical region but gave their final answer for the region as a probability or thought the region was 2 rather than $\leqslant 2$. Candidates, having correctly evaluated the probabilities, were usually successful in finding the probability of a type 1 error.

(c) Again, it is important here that candidates do not quote textbook definitions but answer in the context of the question. It is also important that the two parts to the explanation are clear, i.e., what is concluded and what is the actuality. There were some good answers but many candidates did not answer in context.

(d) The question asked for a reason for the conclusion that the manager made as well as the conclusion itself. Many candidates merely gave a conclusion not realising that a comparison was required, i.e. either to compare 3 with the critical region or to compare $P(X \leqslant 3)$ with 0.05. The conclusion needed to be in context and using non-definite language. In general, this part was well attempted by only a few candidates.

(e) A normal approximation was required here. Many candidates realised this and made a good attempt at the question. Errors included an incorrect, or omission of, a continuity correction and an incorrect probability area.

MATHEMATICS

Paper 9709/62
Probability & Statistics 2 (62)

Key messages

- In all questions, sufficient method must be shown to justify answers.
- Candidates need to work to the required level of accuracy; it is important that accuracy is not lost due to rounding answers to three decimal places rather than three significant figures, or to round too early in the question. If the final answer needs to be given to three significant figures, then all previous numerical answers need to be to at least four significant figures.
- For answers that are required 'in context', quoting general textbook statements will not be sufficient.
- All working should be done in the correct question space of the answer booklet. If answers need to be continued on the Additional page, they must be clearly labelled with the correct question number.
- Candidates should make corrections by crossing through and replacing the work, not by over-writing their answer.

General comments

This was a reasonably well attempted paper, with candidates able to demonstrate their knowledge and application of statistical techniques. Questions that were well attempted were **Questions 2(c), 3(a), 4(c)** and **4(d),** whilst **Question 7(b)** and **Question 5(b)(ii)** were not as well attempted.

Candidates generally presented their answers well and with sufficient working. It is important that sufficient method is shown; for example, if a probability is being calculated using a Poisson distribution, then the term or terms in the Poisson expression must be seen. Candidates must note that conclusion to a hypothesis test must be written in context and with a level of uncertainty in the language used.

Comments on specific questions follow which identify common errors, though it should be noted that there were also many good and fully correct solutions seen as well.

Comments on specific questions

Question 1

This was a reasonably well attempted question. Incorrect values for z were occasionally seen and a common error was to centre the interval around 46 rather than $\frac{46}{200}$. The final answer should have been written as an interval and not as two separate values. Generally, answers were given to the required accuracy level of three significant figures.

Question 2

(a) Many candidates realised that if the random variable W has a Poisson distribution, then $\mathrm{E}(W) = \mathrm{Var}(W)$. However, some candidates did not begin to attempt the question or gave an incorrect relationship.

(b) This part was not as successfully answered as **part (a)**. Some candidates quoted correct formulae for $\mathrm{E}(X)$ and $\mathrm{Var}(X)$, but very few were able to use these to correctly deduce that $(1 - p)$ needed to be close to 1 (so that np and $np(1 - p)$ were approximately equal) and thus p needed to be close to 0. The majority of candidates quoted $np < 5$, n large and p small, or similar conditions rather than using the formulae as requested in the question.

(c) This was a well attempted question. The most common error was to omit the $P(Y = 2)$ term and occasional calculation errors were seen.

Question 3

(a) This question was well attempted with many candidates able to correctly calculate the values required. As has been the case in the past, there was some confusion between the two formulae for the unbiased estimate for the variance, but it was pleasing to note that very few candidates found the biased estimate for the variance.

(b) Some candidates were able to carry out the hypothesis test fully and to give a conclusion which was both in context and using non-definite language. However, many candidates omitted or gave incorrect hypotheses, the comparison statement was sometimes invalid or omitted and the conclusion was not always in context using appropriate non-definite language. Most candidates attempted a two-tailed test, however area comparisons with 0.01, rather than 0.005 after a two-tail test had been correctly declared, were occasionally seen.

Question 4

(a) Some candidates merely stated information that was given in the question (i.e., that the books were received at a constant rate). It was important that the answer was in context, so just stating 'independently' or 'singly' or 'randomly', or 'events should occur independently/singly/randomly', was not sufficient.

(b) This was generally well attempted, though a common error was to give the answer to two significant figures only; 0.044 was often seen as the final answer. This is potentially the result of a confusion between three significant figures and three decimal places. It is important that candidates know how to round to three significant figures, as if only 0.044 is seen here (with no pre-rounded figures) then marks would not be gained.

(c) This part was well attempted. Most candidates used the normal approximation correctly. The most common errors included an incorrect or missing continuity correction, or a probability area that was greater than 0.5 rather than less than 0.5.

(d) This part was also well attempted. Common errors here were using an incorrect value for λ (often 2.5 rather than 5.1 + 2.5) and omission of the $P(3)$ term or omission of '1–….' in the Poisson expression.

Question 5

(a) The majority of candidates scored part marks on this question with very few achieving full marks. Many candidates used $E(X - Y) = 1$ and $\mathrm{Var}(X - Y) = 5$ or equivalent (though some subtracted the variances using 3 – 2 rather than 3 + 2). Many also successfully calculated the probability of $(X - Y) > 2$ but did not identify the other possible case of $(Y - X) > 2$ i.e. $(X - Y) < -2$. Of those who did find the two required probabilities, some multiplied these together rather than adding them.

(b) (i) This part was quite well attempted. The most common error noted was an incorrect value for the variance (using 1.5 rather than 1.5^2) or calculating an incorrect area (<0.5 rather than >0.5).

(ii) This part was not as well attempted. Many candidates stated that T and P had to be independent, and some omitted a response for this part.

Question 6

(a) Most candidates were able to identify $\left(\frac{2}{3}\right)^{10}$ as the required probability but others incorrectly calculated $\left(\frac{1}{3}\right)^{10}$, or $1-\left(\frac{2}{3}\right)^{10}$, or summed until they got their sum of probabilities to be less than $\frac{1}{3}$. As highlighted in the Key messages, this was another case where some candidates did not gain full credit by giving an answer to two significant figures only.

(b) A large number of candidates struggled to set up the correct equation here. Errors included using a mix of both p and q. Candidates who set up the correct equation were usually successful in reaching the correct answer.

Question 7

(a) (i) This was well attempted, with most candidates using the fact that the area of the triangle was equal to 1. However, there was some confusion between the height of the triangle $\left(\frac{1}{2}\right)$ and the value of $k\left(\frac{1}{8}\right)$, with a number of candidates stating that k was $\frac{1}{2}$.

(ii) Again, this was well attempted. Most candidates attempted to integrate x multiplied by their $\mathrm{f}(x)$ using correct limits.

(b) This part was not well attempted, with very few candidates able to reach the correct value for a. Few candidates realised they could use a ratio method (using similar triangles), so the most common method used was to find $\mathrm{g}(w)$ in terms of a and integrate from 0 to 1 (or 1 to a), knowing that this was equal to 0.5. Finding $\mathrm{g}(w)$ in terms of a proved difficult for a large number of candidates.

MATHEMATICS

Paper 9709/63
Probability & Statistics 2 (63)

Key messages

Candidates should be aware that questions on probability density functions may require understanding and use of the pdf properties in different situations. Additionally, candidates are encouraged to make use of diagrams and sketches where appropriate.

General comment

There are several question parts for which the answers are required to be written in context, including in the conclusions to hypothesis tests. Candidates should be prepared to respond to questions in this manner.

Comments on specific questions

Question 1

Many candidates answered this question correctly, integrating $x\mathrm{f}(x)$ between the limits 0 and 1. Some candidates made mistakes when multiplying or when integrating the terms. A few candidates omitted the x and gained no marks.

Question 2

(a) Most candidates found the number for the third member from the list of random numbers. Fewer candidates found the appropriate fourth number. Some candidates suggested 121, but this was a repetition of the second number. Other candidates suggested 473, but this was greater than 264 and so was beyond the highest member's number and so should have been ignored.

(b) Candidates were required to state that these numbers had already been used and so were not random. Some candidates suggested only that they were not random or were biased, which was not sufficient.

Question 3

(a) Some very efficient answers to this question were seen with candidates dealing well with the variance of proportions and the structure of the question. Some candidates found difficulty dealing with $\frac{x}{100}$ and $\left(1-\frac{x}{100}\right)$, and dividing by 100. Other candidates found proportional values (0.36 and 0.64) but did not convert these into the final values for x. Most candidates used the correct z-value of 1.645.

(b) The probability of one of the confidence intervals not containing the true value of p was 0.1. Then the probability of neither of the confidence intervals containing the true value of p was 0.1^2. Some candidates tried squaring 0.9 or some other value. Other candidates tried to use probabilities extracted from a normal distribution.

Question 4

There were two common ways seen of finding the probability. One method involved working with the mass of steel over the 7 days and to find and use N(456.4, 90.72). The main alternative method involved working with the profit over the 7 days and to find and use N(22820, 226800). For either way, it was necessary to standardise and to select the correct probability area. Common errors included not finding the correct variance and working with ‘per day’ instead of ‘per week’. Candidates should be advised that a diagram could be helpful in choosing the correct area, as mentioned in the Key messages.

Question 5

(a) Many candidates found the required unbiased estimates of the population mean and variance. These candidates substituted accurately in the formulae and calculated correctly. A few candidates confused $\frac{1700^2}{50}$ with $\left(\frac{1700}{50}\right)^2$, depending on which of the two formulae for variance was being used.

(b) This significance test required the hypotheses, standardisation, a comparison and a conclusion. The standardisation required the correct use of the sample size of 50. The comparison could be carried out with the critical value for z (2.326) or with the critical probability value (0.01). An alternative method was to find the critical value for t and then compare. The conclusion needed to be stated in context, to not be definite and to have no contradictions.

(c) The question on the circumstances when it would not be necessary to use the Central Limit Theorem was only answered correctly by about a third of candidates.

Question 6

(a) The sample size and the probability led to $np = 2.5$ which indicated the use of a Poisson distribution $\text{Po}(2.5)$. The sum of the probabilities of X being from 0 to 3 was required. It was necessary to list these four individual terms as well as giving the final answer. Many candidates did this correctly.

(b) The value of $\text{E}(X)$ was found from np and the value of $\text{Var}(X)$ from npq. For the second mark, it was required that these two values were referred to in order to explain the appropriateness of the Poisson distribution. Other properties, such as $np < 5$, were not relevant here. Some candidates did not give the answer for the variance accurately enough.

Question 7

(a) This question required the use of the knowledge that the area under a pdf curve is 1. Here, candidates were expected to find the area under the given semicircular graph by using $\frac{\pi r^2}{2}$. Having found the area to be 1, a statement that this indicated that f could be a pdf was required. Some candidates omitted this statement. It was not necessary to state that $\text{f}(x) \geq 0$ for this question.

(b) The angle AOB could be shown to be $\frac{\pi}{4}$ directly by use of the cosine function, but not by other trigonometrical functions unless AB was found first. The further property of pdf functions, i.e., that the probability is given by the specific area under the curve, was then to be used. This area could be found by subtracting the area of the triangle AOB from the area of the sector. This involved careful use of the given lengths in their unusual formats. Answers in decimal form or in terms of π were accepted. Some candidates attempted to use integration to find the area and probability. Many of these candidates were unsuccessful as the integration required was challenging and the formula for the pdf had to be found. However, some marks could be gained for showing correct steps.

Question 8

(a) In this significance test there was a one-tail test and required $\text{P}(X \geq 6)$ using the Poisson distribution $\text{Po}(3.03)$. This tail was found from $1 - \text{P}(X \leq 5)$. This probability (0.0870) then needed to be compared to 0.05 for the 5 per cent significance level. Some candidates used 0.9130 < 0.95 correctly. The conclusion required the same conditions as earlier. Some candidates included an extra term $\mathrm{P}(X = 6)$ incorrectly. This work could be followed through to gain some marks.

(b) The Type I error required $\mathrm{P}(X > 6)$ which could be found from $1 - \mathrm{P}(X \le 6)$ or directly from **part (a)** with the one extra term. For some candidates, work in **part (a)** involving this could be given credit here. Some candidates incorrectly stated that the probability was 0.05.

(c) This statement needed to be expressed in context. Many candidates omitted some of the required parts.

(d) For a Type II error to take place the null hypothesis had to be false. Thus, a new value for the Poisson parameter was required. This had to be expressed in context relating to the mean number of people, the path and the 20 minutes time in the evening.