



## Cambridge International AS & A Level

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**MATHEMATICS**

**9709/31**

Paper 3 Pure Mathematics 3

**May/June 2023**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

1 Solve the equation

$$3e^{2x} - 4e^{-2x} = 5.$$

Give the answer correct to 3 decimal places.

[3]

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2 (a) Sketch the graph of  $y = |2x + 3|$ .

[1]

(b) Solve the inequality  $3x + 8 > |2x + 3|$ .

[3]

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- 4 (a) Show that the equation  $\sin 2\theta + \cos 2\theta = 2 \sin^2 \theta$  can be expressed in the form

$$\cos^2 \theta + 2 \sin \theta \cos \theta - 3 \sin^2 \theta = 0. \quad [2]$$

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- (b) Hence solve the equation  $\sin 2\theta + \cos 2\theta = 2 \sin^2 \theta$  for  $0^\circ < \theta < 180^\circ$ . [4]

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5 The equation of a curve is  $x^2y - ay^2 = 4a^3$ , where  $a$  is a non-zero constant.

(a) Show that  $\frac{dy}{dx} = \frac{2xy}{2ay - x^2}$ . [4]

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(b) The angle between  $BA$  and  $BC$  is  $\theta$ .

Find the exact value of  $\cos \theta$ .

[3]

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(c) Hence find the area of  $ABCD$ , giving your answer in the form  $p\sqrt{q}$ , where  $p$  and  $q$  are integers. [4]

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7 The variables  $x$  and  $y$  satisfy the differential equation

$$\cos 2x \frac{dy}{dx} = \frac{4 \tan 2x}{\sin^2 3y},$$

where  $0 \leq x < \frac{1}{4}\pi$ . It is given that  $y = 0$  when  $x = \frac{1}{6}\pi$ .

Solve the differential equation to obtain the value of  $x$  when  $y = \frac{1}{6}\pi$ . Give your answer correct to 3 decimal places. [8]

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A series of horizontal dotted lines for writing.

8 Let  $f(x) = \frac{3 - 3x^2}{(2x + 1)(x + 2)^2}$ .

- (a) Express  $f(x)$  in partial fractions. [5]

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- (b) Hence find the exact value of  $\int_0^4 f(x) dx$ , giving your answer in the form  $a + b \ln c$ , where  $a$ ,  $b$  and  $c$  are integers. [5]

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9 The constant  $a$  is such that  $\int_0^a xe^{-2x} dx = \frac{1}{8}$ .

(a) Show that  $a = \frac{1}{2} \ln(4a + 2)$ . [5]

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(b) Verify by calculation that  $a$  lies between 0.5 and 1. [2]

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(c) Use an iterative formula based on the equation in (a) to determine  $a$  correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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**10** The polynomial  $x^3 + 5x^2 + 31x + 75$  is denoted by  $p(x)$ .

**(a)** Show that  $(x + 3)$  is a factor of  $p(x)$ . [2]

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**(b)** Show that  $z = -1 + 2\sqrt{6}i$  is a root of  $p(z) = 0$ . [3]

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**Additional Page**

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